Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (7)

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The function f is defined by $f : x \to 3x - 2 \quad for\ x\ \in \mathbb{R}$.

(i) Sketch, in a single diagram, the graphs of $y = f(x) \qquad and\ y = f^{-1}(x)$, making clear the relationship between the two graphs.

The function g is defined by $g : x \to 6x - x^2\ for\ x\ \in \mathbb{R}$.

(ii) Express $g\ f(x)$ in terms of x, and hence show that the maximum value of $g\ f(x)$ is 9.

The function h is defined by

$$h : x \to 6x - x^2\ for\ x\ \geq 3$$

(iii) Express $6x - x^2$ in the form $a - (x - b)^2$, where a and b are positive constants.

(iv) Express $h^{-1}(x)$ in terms of x

Solution

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$f: x \to 3x-2$ for $x \in R$

$f(x) = 3x-2$

(i)

$f(x) = 3x-2$

$y = 3x-2$ $\quad \therefore y = f(x)$

$y+2 = 3x$

$x = \frac{y+2}{3}$ $\quad f^{-1}(y) = x$

$f^{-1}(y) = \frac{y+2}{3}$

replace y by x

$f^{-1}(x) = \frac{x+2}{3}$

$f(x) = y = 3x-2$

$f(1) = 3(1)-2$

$f(1) = 1$

$f(2) = 3(2)-2$

$= 6-2$

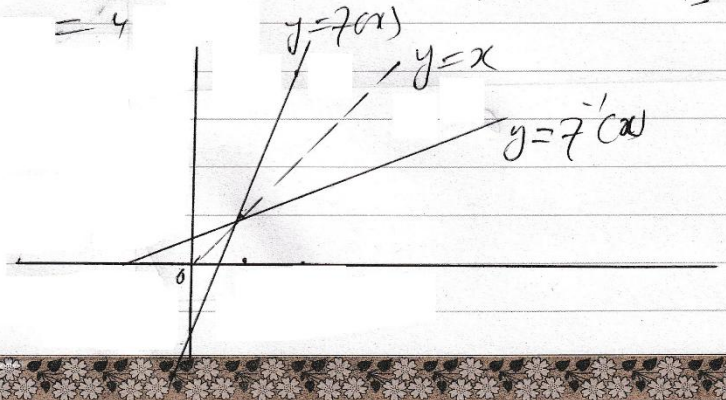
$f(2) = 4$

$f^{-1}(x) = \frac{x+2}{3}$

$f^{-1}(1) = \frac{1+2}{3} = 1$

$f^{-1}(2) = \frac{2+2}{3}$

$f^{-1}(2) = 4/3$



(22)

(ii) $g: x \to 6x - x^2$ for $x \in \mathbb{R}$

$g(x) = 6x - x^2$

As $f(x) = 3x - 2$

$gf(x) = g(3x-2)$ $\quad \because f(x) = 3x-2$

$= 6(3x-2) - (3x-2)^2$

$= 18x - 12 - [9x^2 - 12x + 4]$

$= 18x - 12 - 9x^2 + 12x - 4$

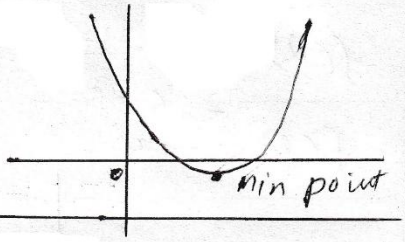
$gf(x) = -9x^2 + 30x - 16$

Knowledge sharing

case1

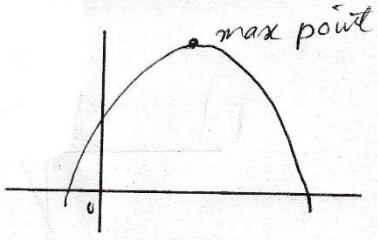
$ax^2 + bx + c$

$a > 0$



$a < 0$

case2



As $a = -9 < 0$ parabola opens downward and we have maximum value.

For calculating maximum value, we have

$$y = \frac{4ac - b^2}{4a}$$

$$y = \frac{4(-9)(-16) - (30)^2}{4(-9)}$$

$$= \frac{576 - 900}{-36} = \frac{-324}{-36}$$

$$y = 9$$

maximum value of $g(x)$ is

$$g(f(x)) = 9$$

(iii)

$h : x \to 6x - x^2$ for $x \geq 3$

$$h(x) = 6x - x^2$$

$$= -x^2 + 6x$$

$$= -(x^2 - 6x)$$

$$= -(x^2 - 2(3)x)$$

$$= -(x^2 - 2(3)x + (3)^2 - (3)^2)$$

$$= -(x^2 - 2(3)(x) + (3)^2) + (3)^2$$

$$= -(x-3)^2 + 9$$

$$= 9 - (x-3)^2 \quad \text{Ans}$$

(iv) As $h(x) = 9-(x-3)^2$ from (iii)

$y = 9-(x-3)^2 \Rightarrow y = h(x)$

$(x-3)^2 = 9-y$

$x-3 = \pm\sqrt{9-y}$

$x-3 = \sqrt{9-y} \Rightarrow x \geq 3$

$x = 3+\sqrt{9-y}$

$h^{-1}(y) = 3 + \sqrt{9-y}$

$h^{-1}(x) = 3 + \sqrt{9-x}$

$h^{-1}(x)$ exist only for $x \geq 3$