Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (8)

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The function f is defined by $f : x \to 2x^2 - 12x + 13 \ \ for\ 0 \leq x \leq A$
where A is a constant.

(i) Express $f(x)$ in the form $a\ (x + b)^2 + c$, where a, b and c are constants.

(ii) State the value of A for which the graph of $y = f(x)$ has a line of symmetry.

(iii) When A has this value, find the range of f.

The function g is defined by $g : x \to 2x^2 - 12x + 13 \ \ for\ x \geq 4$.

(iv) Explain why g has an inverse.

(v) Obtain an expression, in terms of x, for $g^{-1}(x)$.

Solution

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$f: x \to 2x^2 - 12x + 13$ for $0 \le x \le A$

(i)

$$f(x) = 2x^2 - 12x + 13$$

$$= 2(x^2 - 6x) + 13$$

$$= 2(x^2 - 2(3)x) + 13$$

By completing square

$$= 2(x^2 - 2(3)x + (3)^2 - (3)^2) + 13$$

$$= 2(x^2 - 2(3)x + (3)^2) - 2(3)^2 + 13$$

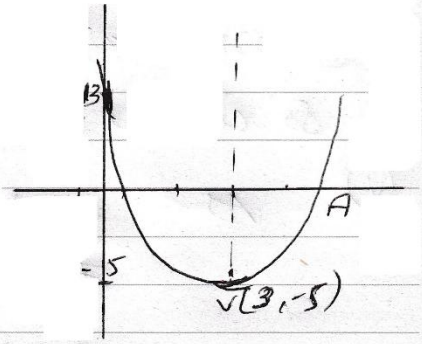
$$= 2(x-3)^2 - 18 + 13$$

$$= 2(x-3)^2 - 5$$

(ii) From part (i)

$f(x) = 2(x-3)^2 - 5$

vertex $(3, -5)$



∴ y has line of symmetry at $x = 3$

As $x = 3$ is line of symmetry, so $A = 6$

∴ $0 \le x \le A$

(iii) As $f: x \to 2x^2 - 12x + 13$ for $0 \le x \le 6$

$\therefore A = 6$

part(i)

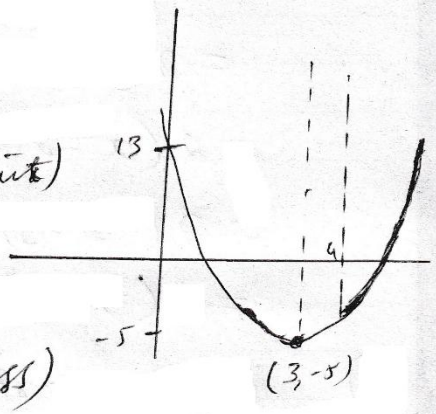
from the graph in part(ii)

range of $f(x)$ is $-5 \le f(x) \le 13$.

(iv) $g: x \to 2x^2 - 12x + 13$ for $x \ge 4$

for $g(x)$, $x \ge 4$

so graph is on right side of $x = 3$ (min point)



Therefore $g(x)$ is a one-one function

(horizontal line test pass)

so it has inverse.

(v) As $g(x) = 2x^2 - 12x + 13$

from part(i)

$g(x) = 2(x-3)^2 - 5$

$\Rightarrow y = 2(x-3)^2 - 5 \quad \therefore g(x) = y$

$$y+5 = 2(x-3)^2$$

$$2(x-3)^2 = y+5$$

$$(x-3)^2 = \frac{y+5}{2}$$

$$x-3 = \pm\sqrt{\frac{y+5}{2}}$$

$$x-3 = +\sqrt{\frac{y+5}{2}}$$ (due to right side of axis of symmetry)

$$x = 3+\sqrt{\frac{y+5}{2}}$$

$$f^{-1}(y) = 3+\sqrt{\frac{y+5}{2}}$$ $\because x = f^{-1}(y)$

replacing y by x

$$f^{-1}(x) = 3+\sqrt{\frac{x+5}{2}}$$