

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (12)

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Question No (12)

The function f is defined by

$$f(x) = x^2 - 4x + 7 \text{ for } x > 2.$$

- (i) Express $f(x)$ in the form $(x - a)^2 + b$ and hence state the range of f .
- (ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

The function g is defined by

$$g(x) = x - 2 \text{ for } x > 2.$$

. The function h is such that $f = hg$ and the domain of h is $x > 0$.

- (iii) Obtain an expression for $h(x)$.

Solution

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$$f(x) = x^2 - 4x + 7 \quad \text{for } x \geq 2$$

(i)

$$f(x) = x^2 - 4x + 7$$

$$f(x) = x^2 - 2(2)x + 7$$

$$f(x) = x^2 - 2(2)x + (2)^2 - (2)^2 + 7 \quad \left(\begin{array}{l} \text{completing} \\ \text{square} \end{array} \right)$$

$$= (x-2)^2 - 4 + 7$$

$$f(x) = (x-2)^2 + 3$$

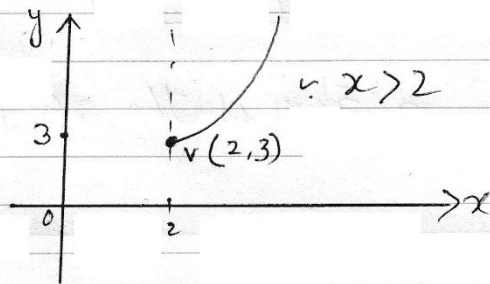
vertex (2, 3)

As in $f(x) = x^2 - 4x + 7$

$$a = 1 > 0$$

parabola $\uparrow$ or down and
will have minimum value

compare
with
 $ax^2 + bx + c$



$\therefore$ range of f is $f(x) \geq 3$

(ii)

$$f(x) = x^2 - 4x + 7$$

$$f(x) = (x-2)^2 + 3$$

From part (i)

$$\Rightarrow y = (x-2)^2 + 3 \quad \because f(x) = y$$

$$y-3 = (x-2)^2$$

$$(x-2)^2 = y-3$$

$$x-2 = \pm \sqrt{y-3}$$

$$x-2 = +\sqrt{y-3}$$

$= x > 2$ it's
+ve

$$x = 2 + \sqrt{y-3}$$

$$\bar{f}'(y) = 2 + \sqrt{y-3}$$

$$\because x = \bar{f}'(y)$$

replacing y by x

$$\bar{f}'(x) = 2 + \sqrt{x-3}$$

Knowledge sharing

domain of $\bar{f}'(x)$ is the range of $f(x)$

from part (i)

range of f is $f(x) \geq 3$

$\Rightarrow$ domain of $\bar{f}'(x)$ is $x \geq 3$

(ii)

$$g(x) = x - 2 \quad \text{for } x > 2$$

$$\text{and } f(x) = (x - 2)^2 + 3$$

Given

$$f = h \circ g$$

$$\Rightarrow f \circ g^{-1} = h$$

$$\Rightarrow h = f \circ g^{-1}$$

$$h(x) = f(x + 2) \quad \because g^{-1}(x) = x + 2$$

$$= ((x + 2) - 2)^2 + 3$$

$$\because f(x) = (x - 2)^2 + 3$$

$$h(x) = x^2 + 3$$

$$g(x) = x - 2$$

$$y = x - 2$$

$$y + 2 = x$$

$$x = y + 2$$

$$g^{-1}(y) = y + 2$$

$$\Rightarrow g^{-1}(x) = x + 2$$