

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (13)

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Question No (13)

The function f is defined by

$$f: x \rightarrow \frac{x+3}{2x-1}, \quad x \in \mathbb{R}, \quad x \neq \frac{1}{2}$$

(i) Show that $ff(x) = x$.

(ii) Hence, or otherwise, obtain an expression for $f^{-1}(x)$.

Solution

$$f: x \rightarrow \frac{x+3}{2x-1}, \quad x \in \mathbb{R}, \quad x \neq \frac{1}{2}$$

$$f(x) = \frac{x+3}{2x-1}$$

② L.H.S

$$f(f(x))$$

$$= \frac{f(x)+3}{2f(x)-1}$$

$$= \frac{\frac{x+3}{2x-1} + 3}{2\left(\frac{x+3}{2x-1}\right) - 1}$$

$$= \frac{x+3 + 3(2x-1)/2x-1}{2(x+3) - 1(2x-1)/2x-1}$$

$$= \frac{x+3 + 6x-3}{2x+6-2x+1}$$

$$= \frac{7x}{7}$$

$$= x$$

$\Rightarrow$ L.H.S = R.H.S

(2)

$$\text{As } f(x) = \frac{x+3}{2x-1}$$

$$\Rightarrow y = \frac{x+3}{2x-1}$$

$$\therefore y = f(x)$$

$$y(2x-1) = x+3$$

$$2xy - y = x+3$$

$$2xy - x = y+3$$

$$x(2y-1) = y+3$$

$$x = \frac{y+3}{2y-1}$$

making x
as subject

$$f^{-1}(y) = \frac{y+3}{2y-1}$$

$$\therefore x = f^{-1}(y)$$

replacing y by x

$$f^{-1}(x) = \frac{x+3}{2x-1}$$