

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (14)

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Question No (14)

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f: x \rightarrow 3x + a$$

$$g: x \rightarrow b - 2x$$

where a and b are constants. Given that $ff(2) = 10$ and $f^{-1}(2) = 3$, find

- (i) the values of a and b ,
- (ii) an expression for $fg(x)$.

Solution

$f: x \rightarrow 3x + a$
 $f(x) = 3x + a$
 and $g: x \rightarrow b - 2x$
 $g(x) = b - 2x$
 Given $ff(2) = 10$ and $f^{-1}(2) = 3$

(i)
 $f(x) = 3x + a$
 $f(2) = 3(2) + a$
 $= 6 + a$
 As $f(f(2)) = 10$
 $\Rightarrow f(6 + a) = 10$
 $3(6 + a) + a = 10$
 $18 + 3a + a = 10$
 $18 + 4a = 10$
 $4a = 10 - 18$
 $4a = -8$
 $a = -2$

As given $f^{-1}(2) = 3$
 $\Rightarrow 2 = f(3)$
 $\Rightarrow 2 = 3(3) + a$
 $2 = 9 + a$
 $a = 2 - 9$
 $a = -7$

$\therefore f(x) = 3x - 2$
 $\therefore g(x) = b - 2x$

$fg(x) = f(b - 2x)$
 $= 3(b - 2x) - 2$
 $= 3b - 6x - 2$

$$a = b - 6$$

$$b = 2 + 6$$

$$b = 8$$

(ii)

$$\text{As } f(x) = 3x - 2$$

$$\text{from part (i)} \\ \therefore a = -2$$

$$g(x) = 8 - 2x$$

$$\therefore b = 8$$

Now

$$f(g(x)) = f(8 - 2x)$$

$$\therefore g(x) = 8 - 2x$$

$$= 3(8 - 2x) - 2$$

$$\therefore f(x) = 3x - 2$$

$$= 24 - 6x - 2$$

$$f(g(x)) = 22 - 6x$$