Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (15)

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Functions f and g are defined by

$$f: x \to 2x + 5 \quad for \ \mathrm{x} \in \mathbb{R},$$

$$g: x \to \frac{8}{x-3} \quad for \ \mathrm{x} \in \mathbb{R}, x \neq 3$$

(i) Obtain expressions, in terms of *x*, for $f^{-1}(x) \ and \ g^{-1}(x)$, stating the value of *x* for which $g^{-1}(x)$ is not defined.

(ii) Sketch the graphs of *y* = f(*x*) and *y* =$f^{-1}(x)$ on the same diagram, making clear the relationship between the two graphs.

(iii) Given that the equation fg(*x*) = 5 – *k x*, where *k* is a constant, has no solutions, find the set of possible values of *k*.

Solution

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$f: x \to 2x+5$ for $x \in R$

$f(x) = 2x+5$

and $g: x \to \frac{8}{x-3}$ for $x \in R, x \neq 3$

$g(x) = \frac{8}{x-3}$

(i)

$f(x) = 2x+5$ $\therefore f(x) = y$

$y = 2x+5$

$y-5 = 2x$

$2x = y-5$

$x = \frac{y-5}{2}$ $\therefore x = f^{-1}(y)$

$f^{-1}(y) = \frac{y-5}{2}$

replacing y by x

$f^{-1}(x) = \frac{x-5}{2}$

NOW $g(x) = \frac{8}{x-3}$

$y = \frac{8}{x-3}$ $\therefore y = f(x)$

$x-3 = \frac{8}{y}$

$x = \frac{8}{y} + 3$ making x subject

$$g^{-1}(y) = \frac{8}{y} + 3$$

$g^{-1}(x) = \frac{8}{x} + 3$, $x \neq 0$ replacing y by x

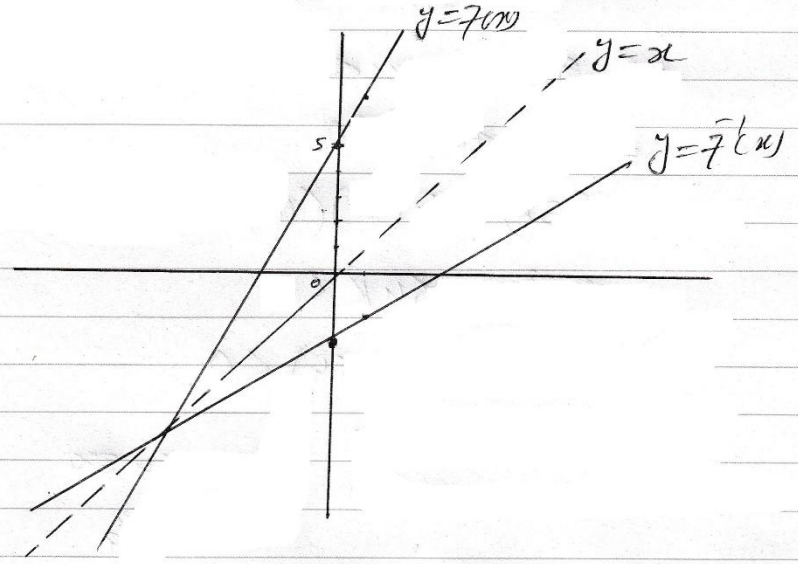
$g^{-1}(x)$ is not defined for $x = 0$

(ii) $y = f(x) = 2x + 5$

$f^{-1}(x) = \frac{x-5}{2}$

x	0	1
y	5	7

x	0	1
$f^{-1}(x)$	-2.5	-2



$f^{-1}(x)$ is the reflection of $f(x)$ in the line of symmetry $y = x$

(iii) Given

$$f(g(x)) = 5 - kx$$

$$f\left(\frac{8}{x-3}\right) = 5 - kx \qquad \because g(x) = \frac{8}{x-3}$$

$$2\left(\frac{8}{x-3}\right) + 5 = 5 - kx \qquad \because f(x) = 2x + 5$$

$$2(8) + 5(x-3) = (x-3)(5-kx)$$

$$16 + 5x - 15 = 5x - kx^2 - 15 + 3kx$$

$$16 + 5x - 15 - 5x + kx^2 + 15 - 3kx = 0$$

$$kx^2 - 3kx + 16 = 0$$

As given equation has no solution

$$\Rightarrow b^2 - 4ac < 0$$

$$\Rightarrow (-3k)^2 - 4(k)(16) < 0$$

$$9k^2 - 64k < 0$$

$$k(9k - 64) < 0$$

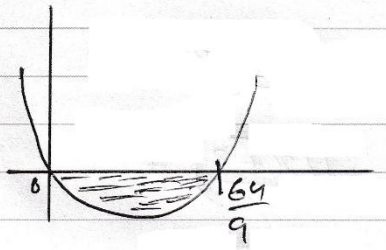
critical values

$k = 0$ $\qquad 9k - 64 = 0$

$k = 64/9$

As $k(9k - 64) < 0$

so we shall have closed interval.



$\therefore \quad 0 < k < \frac{64}{9}$