

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (17)

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Question No (17)

A function f is defined by $f(x) = \frac{5}{1-3x}$, for $x \geq 1$.

(i) Find an expression for $f'(x)$.

(ii) Determine, with a reason, whether f is an increasing function, a decreasing function or neither.

(iii) Find an expression for $f^{-1}(x)$, and state the domain and range of f^{-1} .

Solution

$$f(x) = \frac{5}{1-3x} \quad \text{for } x \geq 1$$

(i)

$$f(x) = \frac{5}{1-3x}$$

$$f(x) = 5(1-3x)^{-1}$$

Diff w.r.t x

$$f'(x) = 5(-1)(1-3x)^{-1-1} \frac{d}{dx}(1-3x)$$

$$= -5(1-3x)^{-2}(-3)$$

$$= 15(1-3x)^{-2}$$

$$f'(x) = \frac{15}{(1-3x)^2}$$

(ii)

$$f'(x) = \frac{15}{(1-3x)^2}$$

As $15 > 0$ and $(1-3x)^2$ is always +ve, so

$$\frac{15}{(1-3x)^2} > 0$$

$\Rightarrow f(x)$ is increasing function.

(iii)

$$\text{As } f(x) = \frac{5}{1-3x}$$

$$y = \frac{5}{1-3x}$$

$$\therefore y = f(x)$$

$$y(1-3x) = 5$$

$$y - 3xy = 5$$

$$-3xy = 5 - y$$

$$3xy = y - 5$$

$$x = \frac{y-5}{3y}$$

$$f^{-1}(y) = \frac{y-5}{3y}$$

$$f^{-1}(x) = \frac{x-5}{3x}$$

replacing
y by x

$$\therefore f^{-1}(x) = \frac{x-5}{3x}$$

$$\text{As } f(x) = \frac{5}{1-3x} \quad \text{for } x \geq 1$$

$$f(1) = \frac{5}{1-3} = -\frac{5}{2} = -2.5$$

$$\text{As } x \rightarrow \infty, f(x) \rightarrow 0$$

$$\therefore \text{range of } f(x) \text{ is } -2.5 \leq f(x) < 0$$

$\Rightarrow$ domain of $\bar{f}(x)$ is $-2.5 \leq x < 0$

As domain of $f(x)$ is $x \geq 1$
 range of $\bar{f}(x)$ is $\bar{f}(x) \geq 1$

Knowledge sharing
 domain of $\bar{f}(x)$ is the range
 of $f(x)$ and vice versa