

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (18)

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Question No (18)

A function f is defined by $f: x \rightarrow 3\cos x - 2$ for $0 \leq x \leq 2$.

- (i) Solve the equation $f(x) = 0$.
- (ii) Find the range of f .
- (iii) Sketch the graph of $y = f(x)$.

A function g is defined by $g: x \rightarrow 3\cos x - 2$ for $0 \leq k \leq 2$.

(iv) State the maximum value of k for which g has an inverse.

(v) Obtain an expression for $g^{-1}(x)$.

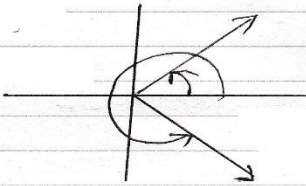
Solution

$f: x \rightarrow 3\cos x - 2$ for $0 \leq x \leq 2\pi$
 $f(x) = 3\cos x - 2$

(i) Given condition
 $f(x) = 0$
 $\Rightarrow 3\cos x - 2 = 0$
 $\Rightarrow 3\cos x = 2$
 $\cos x = \frac{2}{3}$
 Basic angle
 $\cos \alpha = \frac{2}{3}$
 $\alpha = \cos^{-1}\left(\frac{2}{3}\right)$
 $\alpha = 0.841$ radians

So $x = \alpha, 2\pi - \alpha$
 $= 0.841, 2\pi - 0.841$
 $\Rightarrow x = 0.841, 5.45$ radians

(ii) As $f(x) = 3\cos x - 2$
 $y = 3\cos x - 2$
 As for $\cos \theta$ and $\sin \theta$ max and min values.



$$-1 \leq \cos x \leq 1$$

$$-3 \leq 3 \cos x \leq 3$$

$$-3-2 \leq 3 \cos x - 2 \leq 3-2$$

$$-5 \leq 3 \cos x - 2 \leq 1$$

$$-5 \leq f(x) \leq 1$$

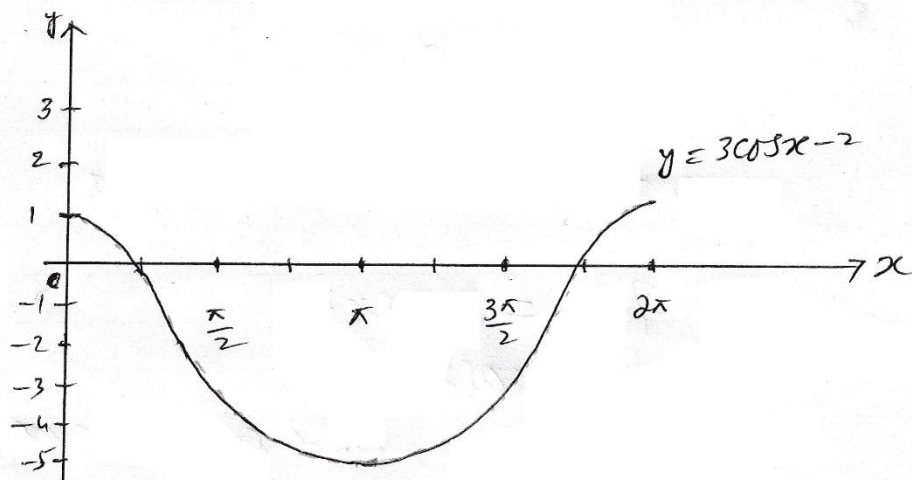
$\therefore$ range of f is $-5 \leq f(x) \leq 1$

(iii)

$$f(x) = 3 \cos x - 2$$

$$y = 3 \cos x - 2 \quad 0 \leq x \leq 2\pi$$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	-2	-5	-2	1



(iv)

$$g: x \rightarrow 3\cos x - 2 \quad \text{for } 0 \leq x \leq \pi$$

$$g(x) = 3\cos x - 2$$

From graph we can see that maximum value of π for which g has inverse is π .

(because up to π horizontal line test pass).

(v)

$$g(x) = 3\cos x - 2$$

$$y = 3\cos x - 2$$

$$\Leftarrow y = g(x)$$

$$y + 2 = 3\cos x$$

$$\frac{y+2}{3} = \cos x$$

$$\cos x = \frac{y+2}{3}$$

$$x = \cos^{-1}\left(\frac{y+2}{3}\right)$$

$$\bar{g}(y) = \cos^{-1}\left(\frac{y+2}{3}\right)$$

replacing y by x

$$\bar{g}(x) = \cos^{-1}\left(\frac{x+2}{3}\right)$$

