Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (19)

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Functions f and g are defined by

$f: x \to 2x - 3,\ x \in \mathbb{R}$.

$g: x \to x^2 + 4x,\ x \in \mathbb{R}$.

(i) Solve the equation $ff(x) = 11$.

(ii) Find the range of g.

(iii) Find the set of values of x for which $g(x) > 12$.

(iv) Find the value of the constant p for which the equation $gf(x) = p$ has two equal roots.

Function h is defined by $h: x \to x^2 + 4x,\ x \geq k$, and it is given that h has an inverse.

(v) State the smallest possible value of k.

(vi) Find an expression for $h^{-1}(x)$.

Solution

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$f: x \to 2x-3 \qquad x \in R$

$f(x) = 2x-3$

and $g: x \to x^2+4x \qquad x \in R$

$g(x) = x^2+4x$

(i)

Given

$ff(x) = 11$

$f(2x-3) = 11 \qquad \because f(x) = 2x-3$

$2(2x-3) - 3 = 11$

$4x - 6 - 3 = 11$

$4x = 11+9$

$4x = 20$

$x = \frac{20}{4}$

$x = 5$

(ii)

$g(x) = x^2+4x$

$g(x) = x^2 + 2(2)x$

$= x^2 + 2(2)x + (2)^2 - (2)^2$

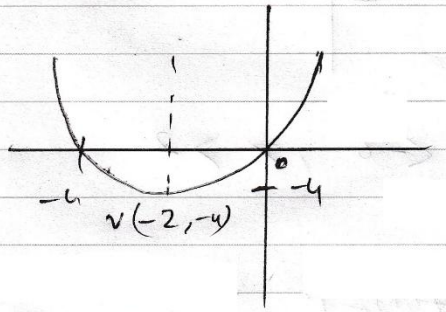
$g(x) = (x+2)^2 - 4$

Vertex $(-2, -4)$

DATE :-

As in $g(x) = x^2 + 4x$

$a = 1 > 0$, so parabola open upward.



$g(x) = x(x+4)$

critical values

$x = 0, x = -4$

so range of $g(x)$ is : $g(x) \geq -4$

(iii) Given

$g(x) > 12$

$\Rightarrow x^2 + 4x > 12$ ← $g(x) = x^2 + 4x$

$x^2 + 4x - 12 > 0$

$x^2 + 6x - 2x - 12 > 0$ factorize

$x(x+6) - 2(x+6) > 0$

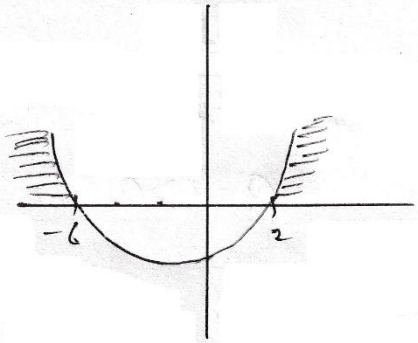
$(x+6)(x-2) > 0$

critical values

$x + 6 = 0, \quad x - 2 = 0$

$x = -6, \quad x = 2$

As $(x+6)(x-2) > 0$



$\therefore \quad x < -6 \;, x > 2$

(iv)

Given $g(x) = p$ has two equal roots

$g(2x-3) = p$ $\quad \because f(x) = 2x-3$

$(2x-3)^2 + 4(2x-3) = p$

$(2x)^2 - 12x + 9 + 8x - 12 = p$

$4x^2 - 12x + 9 + 8x - 12 - p = 0$

$4x^2 - 4x - (p+3) = 0$

As roots are equal

$b^2 - 4ac = 0$

$(-4)^2 - 4(4)(-(p+3)) = 0$

$16 - 16(-(p+3)) = 0$

$16 + 16(p+3) = 0$

$16[1 + p + 3] = 0$

$\Rightarrow 1 + p + 3 = 0$

$p + 4 = 0$

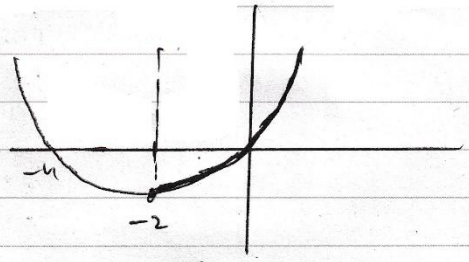
$p = -4$

(v)

$h: x \to x^2 + 4x$ for $x \geq k$

$h(x) = x^2 + 4x$

from part (i)



we see that $g(x)$ is 1-1 function for

$x \geq -2$

$\Rightarrow k = -2$

(vi)

$h(x) = x^2 + 4x$

$\Rightarrow y = x^2 + 4x$

$y = (x+2)^2 - 4$ from part (ii)

$(x+2)^2 = y+4$

$x+2 = \pm\sqrt{y+4}$

$x+2 = \sqrt{y+4}$ + due to required graph on right hand side of symmetry line.

$x = \sqrt{y+4} - 2$

$f^{-1}(y) = \sqrt{y+4} - 2$

replacing y by x

$f^{-1}(x) = \sqrt{x+4} - 2$