

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (20)

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Question No (20)

The function $f: x \rightarrow 6 - 4 \cos\left(\frac{x}{2}\right)$ is defined for $0 \leq x \leq 2\pi$.

- (i) Find the exact value of x for which $f(x) = 4$.
- (ii) State the range of f .
- (iii) Sketch the graph of $y = f(x)$
- (iv) Find an expression for $f^{-1}(x)$.

Solution

$f: x \rightarrow 6 - 4 \cos\left(\frac{1}{2}x\right)$ for $0 \leq x \leq 2\pi$
 $\Rightarrow f(x) = 6 - 4 \cos\left(\frac{x}{2}\right)$
 (1) AS $f(x) = 6 - 4 \cos\left(\frac{x}{2}\right)$
 AS $f(x) = 4$
 $\Rightarrow 4 = 6 - 4 \cos\left(\frac{x}{2}\right)$
 $4 - 6 = -4 \cos\left(\frac{x}{2}\right)$
 $-2 = -4 \cos\frac{x}{2}$
 $\cos\frac{x}{2} = \frac{1}{2}$
 Basic angle
 $\cos\alpha = \frac{1}{2}$
 $\alpha = \cos^{-1}\left(\frac{1}{2}\right)$
 $= \frac{\pi}{3}$

$\cos x$ is true in I and IV quadrant

$$x, 2\pi - x$$

$$\Rightarrow \frac{x}{2} = x, 2\pi - x$$

$$= \pi/3, 2\pi - \pi/3$$

$$x/2 = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x = \frac{2\pi}{3}, \frac{10\pi}{3}$$

As $\frac{10\pi}{3}$ not in $0 \leq x \leq 2\pi$

$$\Rightarrow x = \frac{2\pi}{3}$$

(ii)

From Trigonometry properties of $\cos x$, we have

$$-1 \leq \cos x \leq 1$$

$$-1 \leq \cos x/2 \leq 1$$

multiply by -4

$$4 \geq -4 \cos x/2 \geq -4$$

$$\begin{aligned} & \sim a < b \\ & -a > -b \end{aligned}$$

adding 6

$$6+4 \geq 6-4 \cos x/2 \geq -4+6$$

$$10 \geq 6-4 \cos x/2 \geq 2$$

$$2 \leq 6 - 4 \cos(x/2) \leq 10$$

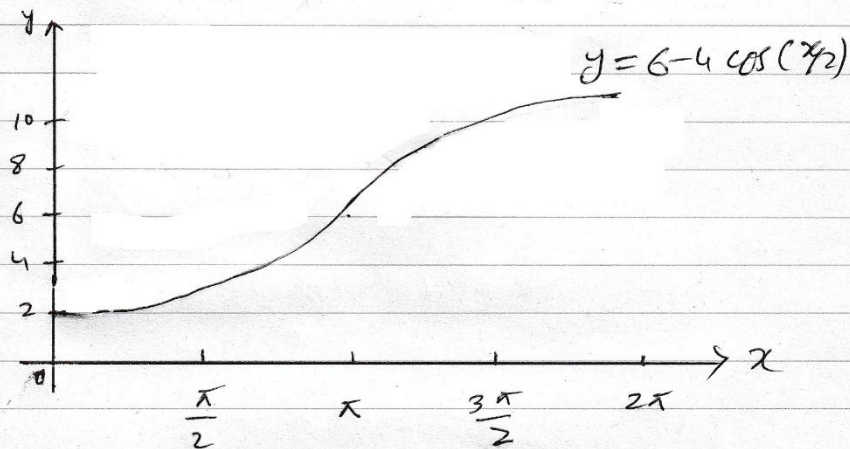
$\therefore$ range of $f(x)$ is

$$2 \leq f(x) \leq 10$$

(iii)

$$y = f(x) = 6 - 4 \cos(x/2) \quad 0 \leq x \leq 2\pi$$

x	0	$\pi/2$	π	$3\pi/2$	2π
y	2	3.2	6	5	10



(iv)

$$\text{As } f(x) = 6 - 4 \cos(x/2)$$

$$y = 6 - 4 \cos(x/2) \quad \Rightarrow f(x) = y$$

$$y - 6 = -4 \cos(x/2)$$

$$6 - y = 4 \cos(x/2)$$

$$\frac{6-y}{4} = \cos(x/2)$$

$$\cos(x/2) = \frac{6-y}{4}$$

$$\frac{x}{2} = \cos^{-1}\left(\frac{6-y}{4}\right)$$

$$x = 2 \cos^{-1}\left(\frac{6-y}{4}\right)$$

$$f'(y) = 2 \cos^{-1}\left(\frac{6-y}{4}\right)$$

replacing y by x

$$f'(x) = 2 \cos^{-1}\left(\frac{6-x}{4}\right)$$