

Cambridge International AS & A Level

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Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (23)

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**Question No (23)**

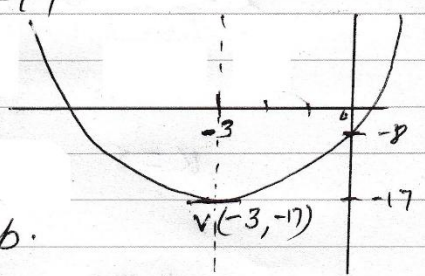
The function  $f$  is defined, for  $x \in \mathbb{R}$ , by  $f: x \rightarrow x^2 + ax + b$ , where  $a$  and  $b$  are constants.

- (i) In the case where  $a = 6$  and  $b = -8$ , find the range of  $f$ .  
 (ii) In the case where  $a = 5$ , the roots of the equation  $f(x) = 0$  are  $k$  and  $-2k$ , where  $k$  is a constant.

Find the values of  $b$  and  $k$ .

- (iii) Show that if the equation  $f(x + a) = a$ , has no real roots, then  $a^2 < 4(b - a)$ .

**Solution**

$f: x \rightarrow x^2 + ax + b$   
 $\Rightarrow f(x) = x^2 + ax + b$   
 (i)  
 $f(x) = x^2 + ax + b \rightarrow \text{①}$   
 at  $a = 6, b = -8$   
 $f(x) = x^2 + 6x - 8$   
 $f(x) = x^2 + 2(3)x - 8$   
 $= x^2 + 2(3)x + (3)^2 - (3)^2 - 8$   
 $= (x+3)^2 - (3)^2 - 8$   
 $= (x+3)^2 - 9 - 8$   
 $f(x) = (x+3)^2 - 17$   
 As  $a = 1 > 0$ , so  
 it will have  
 minimum value  
 parabola  $\uparrow$  a c up.  
  
 $\Rightarrow f(x) \geq -17$   
 (ii)  
 $f(x) = x^2 + ax + b$   
 $f(x) = x^2 + 5x + b$   
 $\therefore a = 5$   
 given

As  $k$  and  $-2k$  are the roots of  $f(x) = 0$

$$\Rightarrow x = k \text{ and } x = -2k$$

$$x - k = 0 \text{ and } x + 2k = 0$$

$$(x - k)(x + 2k) = 0$$

multiplication  
vertically.

$$x(x + 2k) - k(x + 2k) = 0$$

$$x^2 + 2xk - xk - 2k^2 = 0$$

$$x^2 + xk - 2k^2 = 0$$

comparing with

$$x^2 + 5x + b = 0$$

$$\Rightarrow k = 5 \text{ and } b = -2k^2$$

$$k = 5 \text{ and } b = -2(5)^2 = -50$$

$$k = 5 \text{ and } b = -50$$

(iii)

$$\text{As } f(x) = x^2 + ax + b$$

By given condition

$$f(x+a) = a$$

$$(x+a)^2 + a(x+a) + b = a$$

$$x^2 + 2ax + a^2 + ax + a^2 + b = a$$

$$x^2 + 3ax + (2a^2 + b - a) = 0$$

As equation has no real roots

$$b^2 - 4ac < 0$$

$$(3a)^2 - 4(1)(2a^2 + b - a) < 0$$

$$9a^2 - 4(2a^2 + b - a) < 0$$

$$9a^2 - 8a^2 - 4b + 4a < 0$$

$$a^2 - 4b + 4a < 0$$

$$a^2 < 4b - 4a$$

$$a^2 < 4(b - a)$$