

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (26)

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Question No (26)

A function f is defined by $f: x \rightarrow 5 - 2 \sin 2x$ for $0 \leq x \leq \pi$

- (i) Find the range of f
- (ii) Sketch the graph of $y = f(x)$.
- (iii) Solve the equation $f(x) = 6$, giving answers in terms of π .

The function g is defined by $g: x \rightarrow 5 - 2 \sin 2x$ for $0 \leq x \leq k$, where k is a constant.

- (iv) State the largest value of k for which g has an inverse.
- (v) For this value of k , find an expression for $g^{-1}(x)$.

Solution

② $f: x \rightarrow 5 - 2 \sin 2x$ for $0 \leq x \leq \pi$

$f: x \rightarrow 5 - 2 \sin 2x$

$f(x) = 5 - 2 \sin 2x$

From trigonometry function

$-1 \leq \sin x \leq 1$

$-1 \leq \sin 2x \leq 1$

multiply by -2

$2 \geq -2 \sin 2x \geq -2$

$-2 \leq -2 \sin 2x \leq 2$

Adding 5 on both sides.

$5 - 2 \leq 5 - 2 \sin 2x \leq 2 + 5$

$3 \leq 5 - 2 \sin 2x \leq 7$

$3 \leq f(x) \leq 7$ $\because f(x) = 5 - 2 \sin 2x$

$\therefore$ range of f is

$3 \leq f(x) \leq 7$

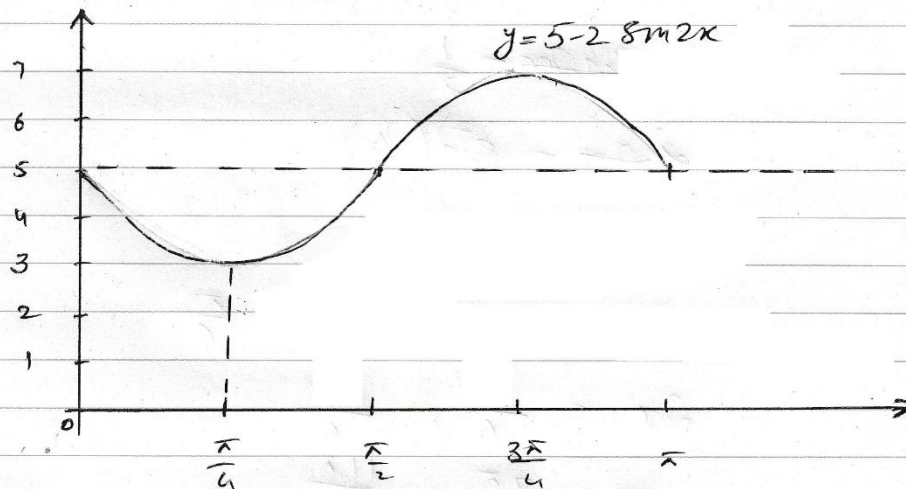
ii

$$y = f(x)$$

$$y = 5 - 2.8 \sin 2x$$

$$0 \leq x \leq \pi$$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(x)$	5	3	5	7	5



iii

Given equation

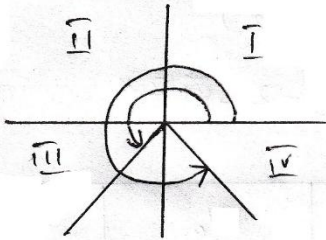
$$f(x) = 6$$

$$\Rightarrow 5 - 2.8 \sin 2x = 6 \quad \because f(x) = 5 - 2.8 \sin 2x$$

$$-2.8 \sin 2x = 6 - 5$$

$$-2.8 \sin 2x = 1$$

$$\sin 2x = -\frac{1}{2}$$



-ve $\sin 2x$ lie in III and IV Quadrant.

Basic angle

$$\sin \alpha = \frac{1}{2}$$

$$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\alpha = \pi/6$$

$$\therefore 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$2x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\therefore x = \frac{7\pi}{12}, \frac{11\pi}{12}$$

(iv)

As $g: x \rightarrow 5 - 2\sin 2x$ for $0 \leq x \leq K$

$$g(x) = 5 - 2\sin 2x$$

From the graph in part (iii),
Horizontal line test pass up to $\pi/4$, so $K = \pi/4$

⑤

$$\text{As } g(x) = 5 - 2 \sin 2x$$

$$\Rightarrow y = 5 - 2 \sin 2x \quad \because g(x) = y$$

$$2 \sin 2x = 5 - y$$

$$\sin 2x = \frac{5-y}{2}$$

$$2x = \sin^{-1} \left(\frac{5-y}{2} \right)$$

$$x = \frac{1}{2} \sin^{-1} \left(\frac{5-y}{2} \right)$$

$$\bar{g}(y) = \frac{1}{2} \sin^{-1} \left(\frac{5-y}{2} \right) \quad \because x = \bar{g}(y)$$

replace y by x

$$\bar{g}(x) = \frac{1}{2} \sin^{-1} \left(\frac{5-x}{2} \right)$$