

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (27)

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Question No (27)

The function f is defined by $f(x) = 3 \tan\left(\frac{x}{2}\right) - 2$, for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

- (i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place.
- (ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1} .
- (iii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

Solution

$$f(x) = 3 \tan\left(\frac{x}{2}\right) - 2, \text{ for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

(i)

Given equation

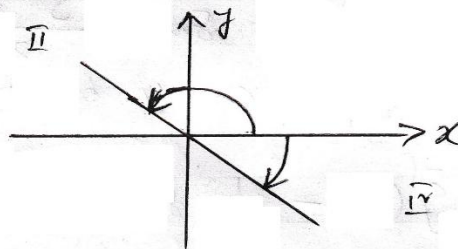
$$f(x) + 4 = 0$$

$$\Rightarrow 3 \tan\left(\frac{x}{2}\right) - 2 + 4 = 0 \quad \because f(x) = 3 \tan\left(\frac{x}{2}\right) - 2$$

$$3 \tan\left(\frac{x}{2}\right) = -2$$

$$\tan\left(\frac{x}{2}\right) = -\frac{2}{3}$$

$\tan$ is -ve in II and IV quadrant



Basic angle

$$\tan \alpha = \frac{2}{3}$$

$$\alpha = \tan^{-1}\left(\frac{2}{3}\right)$$

$$\alpha = 0.588 \text{ rad}$$

Given range $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

we shall take clockwise angle

$$\therefore \frac{x}{2} = -0.588$$

$$x = -1.176 \text{ rad}$$

22

$$f(x) = 3 \tan(x/2) - 2$$

$$y = 3 \tan(x/2) - 2 \quad \text{or } y = f(x)$$

$$y + 2 = 3 \tan(x/2)$$

$$\frac{y+2}{3} = \tan(x/2)$$

$$\Rightarrow \tan(x/2) = \frac{y+2}{3}$$

$$x/2 = \tan^{-1}\left(\frac{y+2}{3}\right)$$

$$x = 2 \tan^{-1}\left(\frac{y+2}{3}\right)$$

$$f^{-1}(y) = 2 \tan^{-1}\left(\frac{y+2}{3}\right) \quad \text{or } x = f^{-1}(y)$$

replace y by x

$$f^{-1}(x) = 2 \tan^{-1}\left(\frac{x+2}{3}\right)$$

as domain of $f(x)$, given is

$$-\pi/2 \leq x \leq \pi/2$$

$$f(x) = 3 \tan(x/2) - 2$$

$$f(-\pi/2) = 3 \tan\left(\frac{1}{2}(-\pi/2)\right) - 2$$

$$= 3 \tan\left(-\frac{\pi}{4}\right) - 2$$

$$f(-\pi/2) = 3(-1) - 2 = -5$$

$$\text{At } x = \pi/2 \quad \therefore -\pi/2 \leq x \leq \pi/2$$

$$f(\pi/2) = 3 \tan\left(\frac{1}{2}(\pi/2)\right) - 2$$

$$= 3 \tan\left(\frac{\pi}{4}\right) - 2$$

$$= 3(1) - 2$$

$$f(\pi/2) = 1$$

range of $f(x)$ is $-5 \leq f(x) \leq 1$

knowledge sharing

Domain of $f^{-1}(x) = \text{range of } f(x)$

$\Rightarrow$ domain of $f^{-1}(x)$ is $-5 \leq x \leq 1$.

(iii)

$$f(x) = 3 \tan(x/2) - 2, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

and

$$f'(x) = 2 \tan\left(\frac{x+2}{3}\right), \quad -5 \leq x \leq 1$$

x	$-\pi/2$	0	$\pi/2$
$f(x)$	-5	-2	1

x	1	-2	0
$f'(x)$	$\pi/2$	0	1.18

