Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (28)

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A function f is defined by $f: x \to 4 - 5x \; for \; x \in \mathbb{R}$.

(i) Find an expression for $f^{-1}(x)$ and find the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$

(ii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$,making clear the relationship between the graphs.

Solution

$f: x \to 4 - 5x$ for $x \in \mathbb{R}$

$f(x) = 4 - 5x$

(i)

$f(x) = 4 - 5x$

$y = 4 - 5x$ $\quad \therefore y = f(x)$

$5x = 4 - y$

$x = \frac{4-y}{5}$

$f^{-1}(y) = \frac{4-y}{5}$ $\quad \therefore x = f^{-1}(y)$

replacing y by x

$f^{-1}(x) = \frac{4-x}{5}$

Now we find the point of intersection of $f(x)$ and $f^{-1}(x)$

$f(x) = f^{-1}(x)$

$4 - 5x = \frac{4-x}{5}$

$5(4 - 5x) = 4 - x$

$20 - 25x = 4 - x$

$20 - 4 = 25x - x$

$16 = 24x$

$x = \frac{16}{24} = \frac{2}{3}$

put $x = \frac{2}{3}$ in

$$y = f(x)$$

$$y = 4 - 5x$$

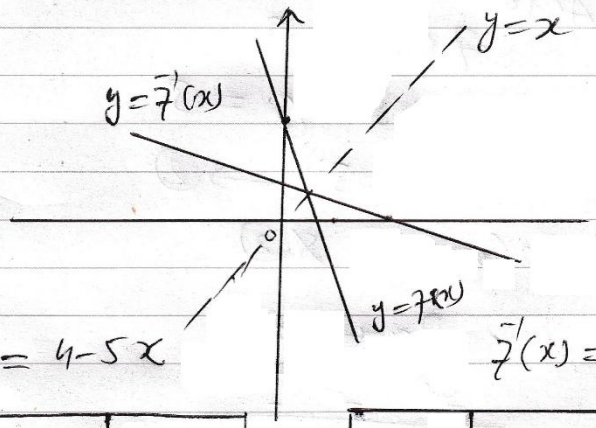
$$= 4 - 5(2/3)$$

$$= 4 - \frac{10}{3}$$

$$y = \frac{12 - 10}{3} = \frac{2}{3}$$

So point of intersection is $\left(\frac{2}{3}, \frac{2}{3}\right)$.

(ii)



x	0	1
$f(x)$	4	−1

x	4	3
$f^{-1}(x)$	0	0.2