

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (29)

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Question No (29)

(a) The function f , defined by $f: x \rightarrow a + b \sin x$ for $x \in \mathbb{R}$, is such that $f\left(\frac{\pi}{6}\right) = 4$ and $f\left(\frac{\pi}{2}\right) = 3$.

(i) Find the values of the constants a and b .

(ii) Evaluate $ff(0)$.

(b) The function g is defined by $f: x \rightarrow c + d \sin x$ for $x \in \mathbb{R}$. The range of g is given by $-4 \leq g(x) \leq 10$.

Find the values of the constants c and d .

Solution

$f: x \rightarrow a + b \sin x$ for $x \in \mathbb{R}$
 $f(x) = a + b \sin x$

(a) (i)
 As $f\left(\frac{\pi}{6}\right) = 4$
 $\Rightarrow f\left(\frac{\pi}{6}\right) = a + b \sin\left(\frac{\pi}{6}\right)$
 $\quad \quad \quad = a + b\left(\frac{1}{2}\right)$
 $4 = a + \frac{b}{2}$ $\therefore f\left(\frac{\pi}{6}\right) = 4$
 $8 = 2a + b$
 $2a + b = 8 \rightarrow \textcircled{1}$

Also $f\left(\frac{\pi}{2}\right) = 3$
 $\Rightarrow a + b \sin \frac{\pi}{2} = 3$
 $a + b = 3 \rightarrow \textcircled{2}$

Solving $\textcircled{1}$ & $\textcircled{2}$

$$\begin{array}{r} 2a + b = 8 \\ a + b = 3 \\ \hline a = 5 \end{array}$$

put $a = 5$ in $\textcircled{2}$
 $5 + b = 3$
 $b = 3 - 5 = -2$
 $b = -2$

Hence $a = 5, b = -2$

(a)

$$\text{As } f(x) = a + b \sin x$$

$$f(x) = 5 - 2 \sin x$$

$$f(0) = 5 - 2 \sin(0)$$

$$= 5 - 2(0)$$

$$f(0) = 5$$

now

$$f(f(0)) = f(5)$$

$$= 5 - 2 \sin 5$$

$$= 5 - (-1.917)$$

$$= 5 + 1.917$$

$$f(f(0)) = 6.917$$

(b)

$$g: x \rightarrow c + d \sin x$$

$$g(x) = c + d \sin x$$

$$-4 \leq g(x) \leq 10$$

least value of function

$$c + d \sin x = -4$$

But $\sin x$ is least at $\sin x = -1$

$$c + d(-1) = -4$$

$$c - d = -4 \rightarrow \textcircled{1}$$

greatest value of $g(x)$

$$c + d \sin x = 10$$

But $\sin x$ is greatest at

$$\sin x = 1$$

$$c + d(1) = 10$$

$$c + d = 10 \rightarrow \textcircled{2}$$

solving $\textcircled{1}$ & $\textcircled{2}$

$$c - d = -4$$

$$c + d = 10$$

$$2c = 6$$

$$c = 3$$

put $c = 3$ in $\textcircled{2}$

$$3 + d = 10$$

$$d = 10 - 3$$

$$d = 7$$

$$\therefore c = 3, d = 7$$