

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (30)

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Question No (30)

The function f is such that $f(x) = a + b \cos x$ for $0 \leq x \leq 2\pi$. It is given that $f\left(\frac{\pi}{3}\right) = 5$ and $f(\pi) = 11$.

(i) Find the values of the constants a and b .

(ii) Find the set of values of k for which the equation $f(x) = k$ has no solution.

Solution

$$f(x) = a + b \cos x \quad \text{for } 0 \leq x \leq 2\pi$$

As $f\left(\frac{\pi}{3}\right) = 5$, $f(\pi) = 11$

(i)

$$f(x) = a + b \cos x$$

$$f\left(\frac{\pi}{3}\right) = a + b \cos\left(\frac{\pi}{3}\right)$$

$$5 = a + b\left(\frac{1}{2}\right) \quad \because f\left(\frac{\pi}{3}\right) = 5$$

$$10 = 2a + b$$

$$2a + b = 10 \rightarrow \textcircled{1}$$

now

$$f(x) = a + b \cos x$$

$$f(\pi) = a + b \cos \pi$$

$$= a + b(-1)$$

$$f(\pi) = a - b$$

$$11 = a - b \quad \because f(\pi) = 11$$

$$a - b = 11 \rightarrow \textcircled{2}$$

solving $\textcircled{1}$ & $\textcircled{2}$

$$\begin{array}{r} 2a + b = 10 \\ a - b = 11 \\ \hline 3a = 21 \\ a = \frac{21}{3} \Rightarrow a = 7 \end{array}$$

put $a=7$ in (2)

$$7 - b = 11$$

$$-b = 11 - 7$$

$$-b = 4$$

$$b = -4$$

$$\therefore a = 7, b = -4$$

(ii)

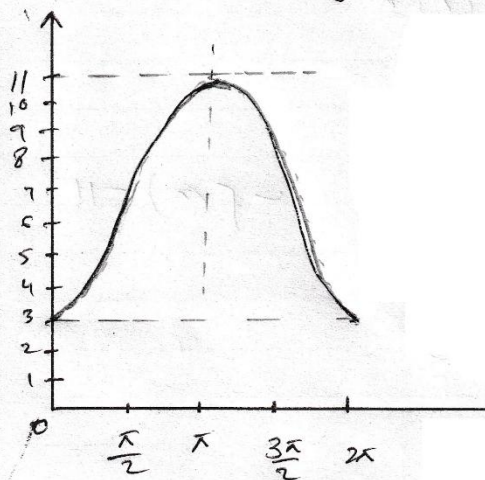
given condition

$$f(x) = k$$

$$\Rightarrow a + b \cos x = k$$

$$7 - 4 \cos x = k \quad \because a = 7, b = -4$$

$$y = 7 - 4 \cos x, \quad y = k$$



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	3	7	11	7	3

From the graph
 $y = f(x) = k$
 has no solution for
 $k < 3$ and $k > 11$

