

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (31)

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Question No (31)

The function f is defined by $f: x \rightarrow 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

(i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x + b)^2$, where a and b are constants

(ii) State the coordinates of the stationary point on the curve $y = f(x)$.

The function g is defined by $g: x \rightarrow 7 - 2x^2 - 12x$ for $x \geq k$.

(iii) State the smallest value of k for which g has an inverse.

(iv) For this value of k , find $g^{-1}(x)$

Solution

$$f: x \rightarrow 7 - 2x^2 - 12x \text{ for } x \in \mathbb{R}$$

$$f(x) = 7 - 2x^2 - 12x$$

(i)

$$\begin{aligned} & 7 - 2x^2 - 12x \\ &= -2x^2 - 12x + 7 \\ &= -2(x^2 + 6x) + 7 \\ &= -2(x^2 + 2(3)(x)) + 7 \\ &= -2(x^2 + 2(3)x + (3)^2 - (3)^2) + 7 \\ &= -2(x + 3)^2 - 2 \times (-6)^2 + 7 \\ &= -2(x + 3)^2 + 18 + 7 \\ &= -2(x + 3)^2 + 25 \\ &= 25 - 2(x + 3)^2 \end{aligned}$$

(ii)

coordinates of the stationary point is vertex $(-3, 25)$

(iii)

$$g: x \rightarrow 7 - 2x^2 - 12x \text{ for } x \geq k$$

$$g(x) = 7 - 2x^2 - 12x$$

smallest value of k is $k = -3$

As $g(x) = 7 - 2x^2 - 12x$ By completing square
 $g(x) = 25 - 2(x+3)^2$ from part (ii)

$$y = 25 - 2(x+3)^2 \quad \Rightarrow \quad g(x) = y$$

$$2(x+3)^2 = 25 - y$$

$$(x+3)^2 = \frac{25-y}{2}$$

$$x+3 = \pm \sqrt{\frac{25-y}{2}}$$

$$x+3 = + \sqrt{\frac{25-y}{2}}$$

$$x = -3 + \sqrt{\frac{25-y}{2}}$$

$$g^{-1}(y) = -3 + \sqrt{\frac{25-y}{2}}$$

replace y by x

$$g^{-1}(x) = -3 + \sqrt{\frac{25-x}{2}}$$

(function is
 R.M.S to
 the axis
 of symmetry
 of $g(x)$)

$$\therefore x = g^{-1}(y)$$