

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (33)

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Question No (33)

The function f is defined by

$$f: x \rightarrow 2x^2 - 12x + 7, \quad x \in \mathbb{R}.$$

- (i) Express $2x^2 - 12x + 7$ in the form $2(x + a)^2 + b$, where a and b are constants.
 (ii) State the range of f .

The function g is defined by $g: x \rightarrow 2x^2 - 12x + 7$ for $x \leq k$.

- (iii) State the largest value of k for which g has an inverse.
 (iv) Given that g has an inverse, find an expression for $g^{-1}(x)$.

Solution**On Next Page**

$$f: x \rightarrow 2x^2 - 12x + 7 \quad \text{for } x \in \mathbb{R}$$

$$f(x) = 2x^2 - 12x + 7$$

(i)

$$\begin{aligned} 2x^2 - 12x + 7 &= 2(x^2 - 6x) + 7 \\ &= 2(x^2 - 2(3)x) + 7 \\ &= 2(x^2 - 2(3)x + (3)^2 - (3)^2) + 7 \\ &= 2(x^2 - 2(3)x + (3)^2) - 2 \times (3)^2 + 7 \\ &= 2(x-3)^2 - 2 \times 9 + 7 \\ &= 2(x-3)^2 - 18 + 7 \\ &= 2(x-3)^2 - 11 \end{aligned}$$

(ii)

given function

$$f(x) = 2x^2 - 12x + 7$$

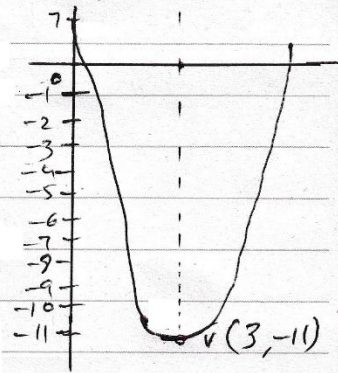
$a = 2 > 0$ so it will have
minimum value. parabola
face up.

from part (i)

$$\begin{aligned} f(x) &= 2x^2 - 12x + 7 \\ &= 2(x-3)^2 - 11 \end{aligned}$$

vertex $(3, -11)$

range of f is $f(x) \geq -11$



(iii)

$$g: x \rightarrow 2x^2 - 12x + 7 \text{ for } x \leq k$$

$$g(x) = 2x^2 - 12x + 7$$

From the graph of part (ii), we check horizontal line test pass when largest value of $k = 3$,

$$\text{so } k = 3$$

(iv)

$$g(x) = 2x^2 - 12x + 7$$

from part (i)

$$g(x) = 2(x-3)^2 - 11$$

$$y = 2(x-3)^2 - 11$$

$$\hookrightarrow y = g(x)$$

$$y + 11 = 2(x-3)^2$$

$$2(x-3)^2 = y + 11$$

$$(x-3)^2 = \frac{y+11}{2}$$

$$x-3 = \pm \sqrt{\frac{y+11}{2}}$$

$$x-3 = -\sqrt{\frac{y+11}{2}}$$

$$\hookrightarrow x \leq 3$$

$$x = 3 - \sqrt{\frac{y+11}{2}}$$

$$\bar{g}(y) = 3 - \sqrt{\frac{y+11}{2}}$$

$$\hookrightarrow x = \bar{g}(y)$$

$$\bar{g}(x) = 3 - \sqrt{\frac{x+11}{2}}$$

replace y by x

