Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (34)

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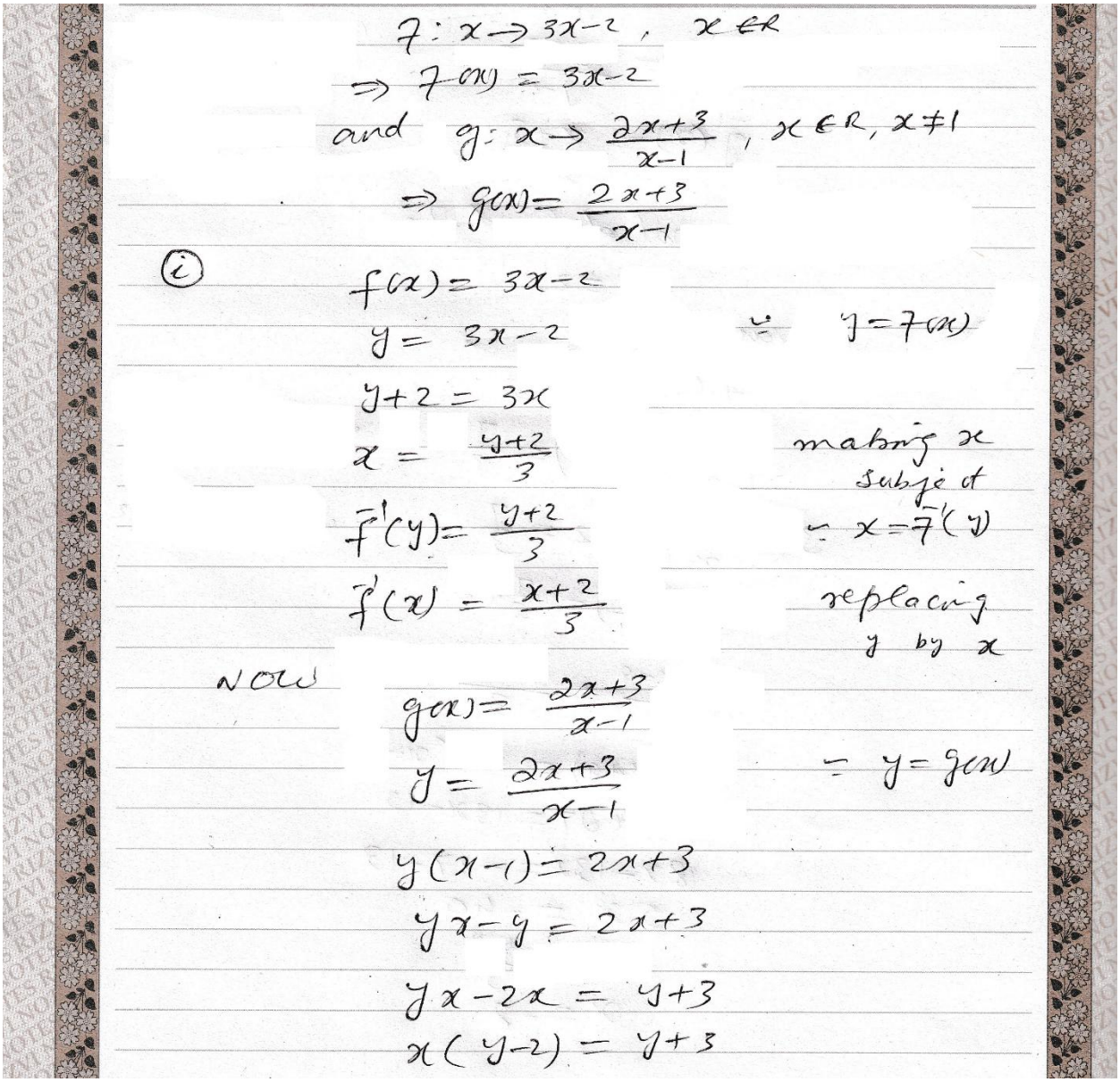
Functions f and g are defined by $f: x \to 3x - 2$, $\quad x \in \mathbb{R}$,

$$g: x \to \frac{2x+3}{x-1}, \quad x \in \mathbb{R}, x \neq 1$$

(i) Obtain expressions for $f^{-1}(x)$ *and* $g^{-1}(x)$,stating the value of x for which $g^{-1}(x)$ is not defined.

(ii) Solve the equation $f\ g(x) = \frac{7}{3}$

Solution



$$x = \frac{y+3}{y-2}$$

$$g^{-1}(y) = \frac{y+3}{y-2} \quad \therefore \quad x = g^{-1}(y)$$

$$g^{-1}(x) = \frac{x+3}{x-2} \quad \text{replace } y \text{ by } x$$

$g^{-1}(x)$ is not defined for

$$x = 2$$

(ii) Given condition

$$f(g(x)) = \frac{7}{3}$$

$$f\left(\frac{2x+3}{x-1}\right) = \frac{7}{3} \quad \therefore g(x) = \frac{2x+3}{x-1}$$

$$3\left(\frac{2x+3}{x-1}\right) - 2 = \frac{7}{3} \quad \therefore f(x) = 3x-2$$

$$3\left(\frac{2x+3}{x-1}\right) = \frac{7}{3} + 2$$

$$\frac{6x+9}{x-1} = \frac{7+6}{3} = \frac{13}{3}$$

$$3(6x+9) = 13(x-1)$$

$$18x + 27 = 13x - 13$$

$$18x - 13x = -27 - 13$$

$$5x = -40$$

$$x = -\frac{40}{5}$$

$$x = -8$$