

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (34)

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Functions f and g are defined by $f: x \rightarrow 3x - 2$, $x \in \mathbb{R}$,

$$g: x \rightarrow \frac{2x+3}{x-1}, \quad x \in \mathbb{R}, x \neq 1$$

(i) Obtain expressions for $f^{-1}(x)$ and $g^{-1}(x)$, stating the value of x for which $g^{-1}(x)$ is not defined.

(ii) Solve the equation $f \circ g(x) = \frac{7}{3}$

Solution

$f: x \rightarrow 3x - 2, \quad x \in \mathbb{R}$
 $\Rightarrow f(x) = 3x - 2$
 and $g: x \rightarrow \frac{2x+3}{x-1}, \quad x \in \mathbb{R}, x \neq 1$
 $\Rightarrow g(x) = \frac{2x+3}{x-1}$

(i) $f(x) = 3x - 2$
 $y = 3x - 2$
 $y + 2 = 3x$
 $x = \frac{y+2}{3}$
 $f^{-1}(y) = \frac{y+2}{3}$
 $f^{-1}(x) = \frac{x+2}{3}$

making x subject
 $\therefore x = f^{-1}(y)$
 replacing y by x
 $\therefore y = f(x)$

now $g(x) = \frac{2x+3}{x-1}$
 $y = \frac{2x+3}{x-1}$
 $y(x-1) = 2x+3$
 $yx - y = 2x+3$
 $yx - 2x = y+3$
 $x(y-2) = y+3$

$$x = \frac{y+3}{y-2}$$

$$\bar{g}(y) = \frac{y+3}{y-2}$$

$$\bar{g}(x) = \frac{x+3}{x-2}$$

$$\therefore x = \bar{g}(y)$$

replace y by x

$\bar{g}(x)$ is not defined for

$$x=2$$

(ii)

Given condition

$$f(g(x)) = \frac{7}{3}$$

$$f\left(\frac{2x+3}{x-1}\right) = \frac{7}{3}$$

$$\therefore g(x) = \frac{2x+3}{x-1}$$

$$3\left(\frac{2x+3}{x-1}\right) - 2 = \frac{7}{3}$$

$$\therefore f(x) = 3x-2$$

$$3\left(\frac{2x+3}{x-1}\right) = \frac{7}{3} + 2$$

$$\frac{6x+9}{x-1} = \frac{7+6}{3} = \frac{13}{3}$$

$$3(6x+9) = 13(x-1)$$

$$18x+27 = 13x-13$$

$$18x-13x = -27-13$$

$$5x = -40$$

$$x = -\frac{40}{5}$$

$$x = -8$$

