Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (35)

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Functions f and g are defined by

$$f(x) = 2\,x^2 + 8x + 1, for\ \ x\ \in\ \mathbb{R},$$

$$g(x) = 2x - k \quad for\ \ x\ \in\ \mathbb{R},$$

, where k is a constant.

(i) Find the value of k for which the line $y = g(x)$ is a tangent to the curve $y = f(x)$.

(ii) In the case where k=−9, find the set of values of x for which $f(x) < g(x)$.

(iii) In the case where k=−1, find $g^{-1}f(x)$ and solve the equation $g^{-1}f(x) = 0$

(iv) Express $f(x)$ in the form $2\,(x + a)^2 + b$,where a and b are constants, and hence state the least value of $f(x)$.

Solution

$f(x) = 2x^2 + 8x + 1$ for $x \in R$

$g(x) = 2x - k$ for $x \in R$

(i)

$y = 2x^2 + 8x + 1 \rightarrow (1)$

and line

$y = 2x - k \longrightarrow (2)$

As line is tangent to the curve, so they intersect at one point.

From (1) & (2)

$2x^2 + 8x + 1 = 2x - k$

$2x^2 + 8x + 1 - 2x + k = 0$

$2x^2 + 6x + k + 1 = 0$

As line is tangent to curve

$b^2 - 4ac = 0$

$(6)^2 - 4(2)(k+1) = 0$

$36 - 8(k+1) = 0$

$36 - 8k - 8 = 0$

$28 - 8k = 0$

$-8k = -28$

$k = \frac{28}{8}$

$k = 3.5$

(ii) given condition

$f(x) < g(x)$

$2x^2+8x+1 < 2x-k$

$2x^2+8x+1 < 2x-(-9)$ $\because k=-9$

$2x^2+8x+1 < 2x+9$

$2x^2+8x+1-2x-9<0$

$2x^2+6x-8<0$

$2(x^2+3x-4)<0$

$x^2+3x-4<0$ $\because 2>0$

$x^2+4x-x-4<0$ factorization

$x(x+4)-1(x+4)<0$

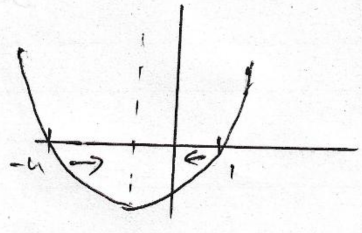
$(x+4)(x-1)<0$

critical values

$x+4=0,\ x-1=0$

$x=-4,\ x=1$

As in $x^2+3x-4<0$, $a=1>0$, so parabola face up



As $(x+4)(x-1)<0$
So, we shall take closed interval

$-4<x<1$

(iii)

$$g(x) = 2x - k$$

$$g(x) = 2x - (-1) \quad \because k = -1$$

$$g(x) = 2x + 1$$

$$y = 2x + 1 \quad \because y = f(x)$$

$$y - 1 = 2x$$

$$\frac{y-1}{2} = x$$

$$\Rightarrow x = \frac{y-1}{2}$$

$$g^{-1}(y) = \frac{y-1}{2} \quad \because x = g^{-1}(y)$$

$$g^{-1}(x) = \frac{x-1}{2} \quad \text{replacing } y \text{ by } x$$

Given condition

$$g^{-1}(f(x)) = \frac{f(x) - 1}{2}$$

$$= \frac{2x^2 + 8x + 1 - 1}{2}$$

$$= \frac{2x^2 + 8x}{2}$$

$$= \frac{2(x^2 + 4x)}{2}$$

$$= x^2 + 4x$$

As $g'(f(x)) = 0$ (given)

$\Rightarrow x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0, \quad x + 4 = 0$

$x = 0, \quad x = -4$

(iv)

$f(x) = 2x^2 + 8x + 1$

$= 2(x^2 + 4x) + 1$

$= 2(x^2 + 2(2)x) + 1$

$= 2(x^2 + 2(2)x + (2)^2 - (2)^2) + 1$

$= 2(x^2 + 2(2)x + (2)^2) - 2(2)^2 + 1$

$= 2(x+2)^2 - 8 + 1$

$f(x) = 2(x+2)^2 - 7$

vertex $(-2, -7)$

As $a = 2 > 0$, so it will have least value.

Least value of $f(x)$ is

$f(x) = -7$