

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (36)

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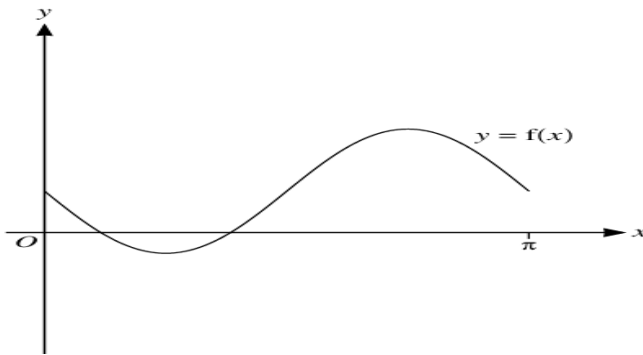
Functions f and g are such that

$$f(x) = 2 - 3 \sin 2x \text{ for } 0 \leq x \leq \pi,$$

$$g(x) = -2f(x) \text{ for } 0 \leq x \leq \pi.$$

(a) State the ranges of f and g .

The diagram below shows the graph of $y = f(x)$.



(b) Sketch, on this diagram, the graph of $y = g(x)$.

The function h is such that $h(x) = g(x + \pi)$ for $-\pi \leq x \leq 0$.

(c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$.

Solution

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$$f(x) = 2 - 3 \sin 2x$$

$$\text{for } 0 \leq x \leq \pi$$

$$g(x) = -2 f(x)$$

$$\text{for } 0 \leq x \leq \pi$$

(a)

$$\text{As } -1 \leq \sin x \leq 1$$

By trigonometry
law

$$-1 \leq \sin 2x \leq 1$$

$$3 \geq -3 \sin 2x \geq -3$$

multiply by -3

$$-3 \leq -3 \sin 2x \leq 3$$

$$2 - 3 \leq 2 - 3 \sin 2x \leq 2 + 3$$

$$-1 \leq 2 - 3 \sin 2x \leq 5$$

$$-1 \leq f(x) \leq 5$$

$$\therefore f(x) = 2 - 3 \sin 2x$$

so range of $f(x)$ is

$$-1 \leq f(x) \leq 5$$

now

$$g(x) = -2 f(x)$$

$$\text{for } 0 \leq x \leq \pi$$

$$g(x) = -2(2 - 3 \sin 2x)$$

$$= -4 + 6 \sin 2x$$

$$g(x) = 6 \sin 2x - 4$$

$$\text{As } -1 \leq \sin x \leq 1$$

$$-1 \leq \sin 2x \leq 1$$

$$-6 \leq 6 \sin 2x \leq 6$$

multiply
by 6

$$-6-4 \leq 6\sin 2x-4 \leq 6-4$$

$$-10 \leq 6\sin 2x-4 \leq 2$$

$$-10 \leq g(x) \leq 2$$

range of $g(x)$ is $-10 \leq g(x) \leq 2$

(b)

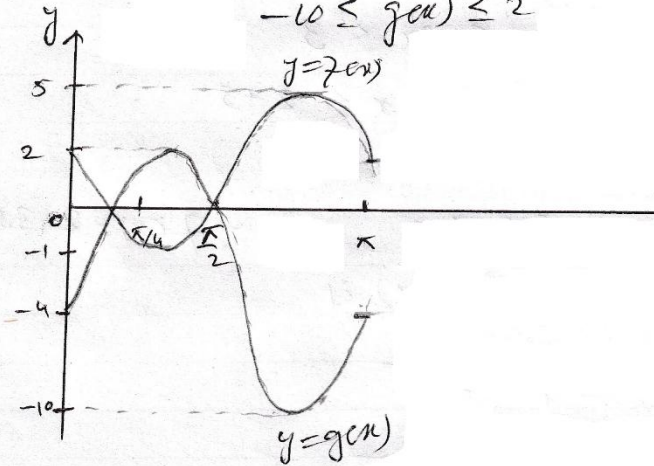
range of $f(x)$

$$-1 \leq f(x) \leq 5$$

from part (a)

and range of $g(x)$

$$-10 \leq g(x) \leq 2$$



(c)

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leq x \leq 0$$

$$f(x) = 2 - 3\sin 2x, \text{ As } g(x) = 6\sin 2x - 4$$

$$f(x) = 2 - 38\pi^2 x \xrightarrow{-2} g(x) = 68\pi^2 x - 4$$

reflect in x -axis as due to multiply
 y -coordinate by -2 reflect in x -axis
 and stretching factor 2 along
 y -axis.

Translate horizontally π units
 towards $-ve$ x -axis due to $x + \pi \Rightarrow$
 $x = -\pi$