Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (37)

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The function f is defined for $x \in \mathbb{R}$ by

$$f: x \to a - 2x ,$$

Where a is a constant.

(a) Express $ff(x)$ and $f^{-1}(x)$ in terms of a and x.

(b) Given that $ff(x) = f^{-1}(x)$,find x in terms of a.

Solution

$f: x \to a - 2x$

$f(x) = a - 2x$

$f(x) = a - 2x$

$f(f(x)) = a - 2(f(x))$

$= a - 2(a - 2x)$

$= a - 2a + 4x$

$ff(x) = 4x - a$

Now

$f(x) = a - 2x$

$y = a - 2x$ $\quad \therefore y = f(x)$

$2x = a - y$

$x = \frac{a - y}{2}$ making x subject $\quad \therefore x = f^{-1}(y)$

$f^{-1}(y) = \frac{a - y}{2}$

$f^{-1}(x) = \frac{a - x}{2}$ replacing y by x

(b)

As given

$ff(x) = f^{-1}(x)$

$\Rightarrow 4x - a = \frac{a - x}{2}$

$$2(4x-a) = a-x$$

$$8x - 2a = a - x$$

$$8x + x = a + 2a$$

$$9x = 3a$$

$$x = \frac{3a}{9}$$

$$x = a/3$$