

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (38)

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Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R}, x \neq -1$$

(a) Find the value of $fg(7)$.

(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$.

Solution

$f(x) = 4x - 2$ for $x \in \mathbb{R}$
 $g(x) = \frac{4}{x+1}$ for $x \in \mathbb{R}, x \neq -1$

(a)

$$fg(7) = f\left(\frac{4}{7+1}\right) = f\left(\frac{4}{8}\right) = f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right) - 2 = 2 - 2 = 0$$

(b)

$$f(x) = 4x - 2$$

$$y = 4x - 2$$

$$y + 2 = 4x$$

$$x = \frac{y+2}{4}$$

$$f^{-1}(y) = \frac{y+2}{4}$$

$$f^{-1}(x) = \frac{x+2}{4}$$

now $g(x) = \frac{4}{x+1}$

$y = f(x)$
 making x as subject
 $x = f^{-1}(y)$
 replace y by x

$$y = \frac{4}{x+1}$$

$$\hookrightarrow f(x) = y$$

$$y(x+1) = 4$$

$$yx + y = 4$$

$$yx = 4 - y$$

$$x = \frac{4-y}{y}$$

making x subject

$$\bar{g}'(y) = \frac{4-y}{y}$$

$$\bar{g}'(x) = \frac{4-x}{x}$$

replace y by x

Given condition

$$\bar{f}'(x) = \bar{g}'(x)$$

$$\frac{x+2}{4} = \frac{4-x}{x}$$

$$x(x+2) = 4(4-x)$$

$$x^2 + 2x = 16 - 4x$$

$$x^2 + 2x + 4x - 16 = 0$$

$$x^2 + 6x - 16 = 0$$

Factorization

$$x^2 + 8x - 2x - 16 = 0$$

$$x(x+8) - 2(x+8) = 0$$

$$(x+8)(x-2) = 0$$

$$x-2=0$$

$$x=2$$

$$x+8=0,$$

$$x=-8$$

