

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (39)

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Question No (39)

A curve has equation $y = 3 \cos 2x + 2$, $0 \leq x \leq \pi$.

- (a) State the greatest and least values of y .
- (b) Sketch the graph of $y = 3 \cos 2x + 2$, $0 \leq x \leq \pi$.
- (c) By considering the straight line $y = kx$, $0 \leq x \leq \pi$, where k is a constant, state the number of solutions of the equation $3 \cos 2x + 2 = kx$ for $0 \leq x \leq \pi$ in each of the following cases.
- (i) $k = -3$
- (ii) $k = 1$
- (iii) $k = 3$

Functions f , g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3 \cos 2x + 2,$$

$$g(x) = f(2x) + 4,$$

$$h(x) = 2f\left(x + \frac{\pi}{2}\right).$$

- (d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$.
- (e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$.

Solution

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$$y = 3 \cos 2x + 2 \quad \text{for } 0 \leq x \leq \pi$$

(a)

$$y = f(x) = 3 \cos 2x + 2$$

As from trigonometry

$$-1 \leq \cos x \leq 1$$

$$-1 \leq \cos 2x \leq 1 \quad \text{multiply by 3}$$

$$-3 \leq 3 \cos 2x \leq 3$$

add 2

$$2 - 3 \leq 2 + 3 \cos 2x \leq 3 + 2$$

$$-1 \leq 3 \cos 2x + 2 \leq 5$$

$$-1 \leq f(x) \leq 5$$

least value = -1

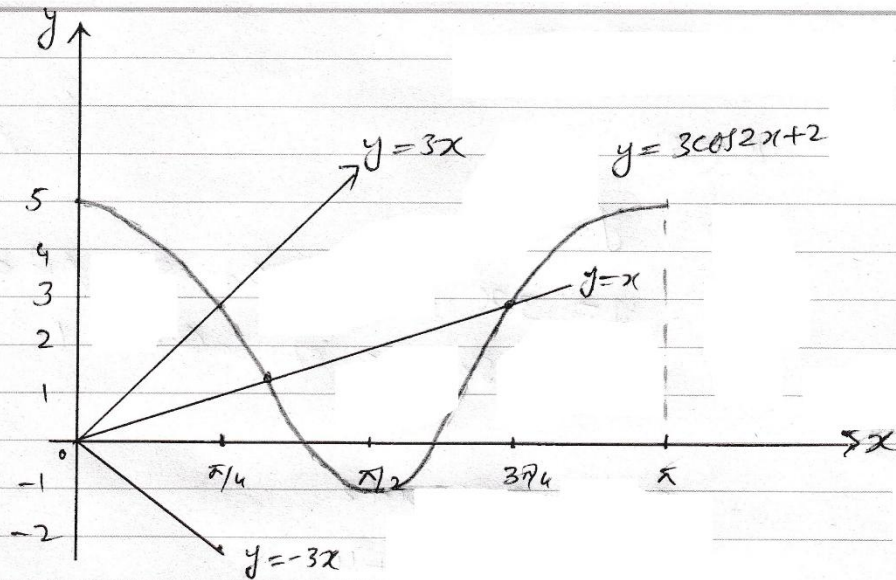
greatest value = 5

(b)

$$y = 3 \cos 2x + 2$$

$$0 \leq x \leq \pi$$

x	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	2	-1	2	5



©

(i)

$$y = kx$$

and $3\cos 2x + 2 = kx$

at $k = -3$

$$\Rightarrow y = -3x$$

From graph $y = -3x$ does not meet the curve,
number of solutions 0.

(ii)

$$k = 1$$

$$\Rightarrow y = x$$

Line $y = x$ cut the graph

at two points, so number of solutions
are 2

(iv) $k=3$

$$\Rightarrow y = 3x$$

Line $y = 3x$ cut the graph at
one point, so no of solutions 1.

(d)

$$f(x) = 3\cos 2x + 2$$

$$g(x) = f(2x) + 4$$

$$h(x) = 27(x + \pi/2)$$

① stretch $y = f(x)$ by factor $\frac{1}{2}$ parallel
to x -axis due to $f(2x)$

② Translation by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ due to $+4$ in $f(2x) + 4$

(e)

$$h(x) = 27(x + \pi/2)$$

① translate $y = f(x)$ by $\begin{pmatrix} -\pi/2 \\ 0 \end{pmatrix}$

② stretch by a factor of 2 parallel to
 y -axis due to $2f(x + \pi/2)$

