

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 3-Coordinate Geometry

Question No (11)

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Question No (11)

The line L_1 passes through the points A (2, 5) and B (10, 9). The line L_2 is parallel to L_1 and passes through the origin. The point C lies on L_2 such that AC is perpendicular to L_2 . Find

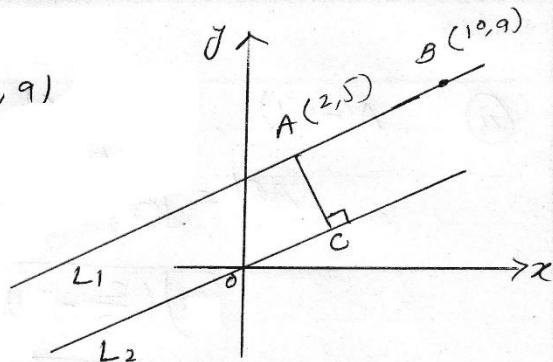
- (i) the coordinates of C,
- (ii) the distance AC.

Solution

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Given data

② $A(2, 5)$ and $B(10, 9)$



$$\begin{aligned}
 \text{Gradient of } AB &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{9 - 5}{10 - 2} \\
 &= \frac{4}{8} \\
 &= \frac{1}{2}
 \end{aligned}$$

Given AC is perpendicular to L_2 ($AC \perp L_2$)

$\therefore$ gradient of AC = -2

Equation of line AC passing through $A(2, 5)$ and gradient -2, we have

$$y - y_1 = -2(x - x_1)$$

$$y-5 = -2(x-2)$$

$$y-5 = -2x+4$$

$$y = -2x+4+5$$

$$y = -2x+9 \rightarrow \textcircled{1}$$

As L_1 is parallel to L_2 ($L_1 \parallel L_2$).

gradient of L_2 is $\frac{1}{2}$ $\therefore$ gradient of L_1 is $\frac{1}{2}$

Equation of line L_2 passing through origin $(0,0)$ and having gradient $\frac{1}{2}$

$$y-y_1 = \frac{1}{2}(x-x_1)$$

$$y-0 = \frac{1}{2}(x-0)$$

$$y = \frac{x}{2} \rightarrow \textcircled{2}$$

now we find point C, solving $\textcircled{1}$ & $\textcircled{2}$,
put Equation $\textcircled{2}$ in $\textcircled{1}$

$$\frac{x}{2} = -2x+9$$

$$x = -4x+18$$

$$x+4x = 18$$

$$5x = 18$$

$$x = \frac{18}{5}$$

put $x = \frac{18}{5}$ in equation ②

$$y = \frac{1}{2} \left(\frac{18}{5} \right)$$

$$y = \frac{9}{5}$$

So coordinates of C are $\left(\frac{18}{5}, \frac{9}{5} \right)$.

(ii)

As A(2, 5) and C $\left(\frac{18}{5}, \frac{9}{5} \right)$

Distance formula

$$|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(\frac{18}{5} - 2 \right)^2 + \left(\frac{9}{5} - 5 \right)^2}$$

$$= \sqrt{\left(\frac{18 - 10}{5} \right)^2 + \left(\frac{9 - 25}{5} \right)^2}$$

$$= \sqrt{\left(\frac{8}{5} \right)^2 + \left(\frac{-16}{5} \right)^2}$$

$$= \sqrt{\frac{64}{25} + \frac{256}{25}}$$

$$= \sqrt{\frac{64 + 256}{25}} = \frac{\sqrt{320}}{5} = \frac{\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5}}{5}$$

$$= \frac{8\sqrt{5}}{5}$$

