

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

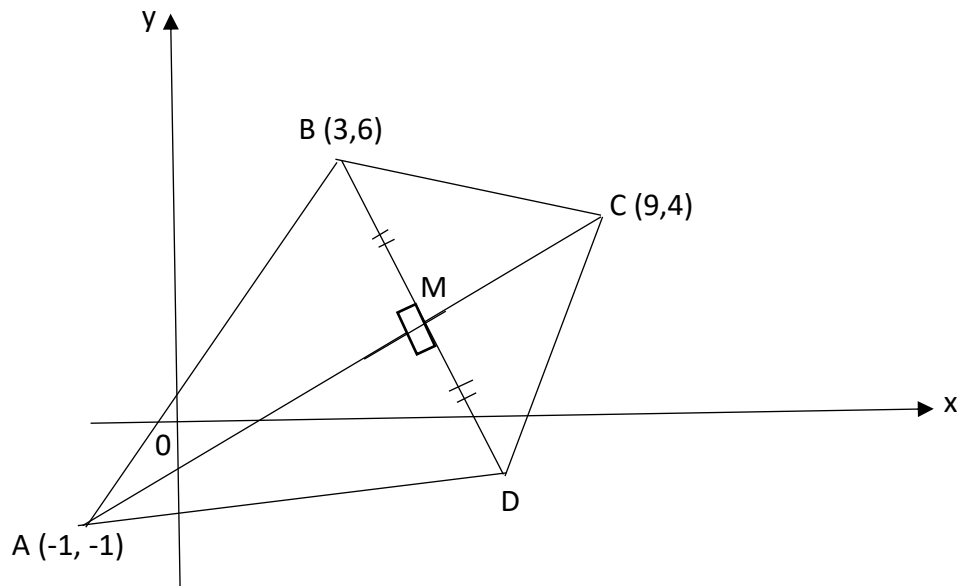
Topic 3-Coordinate Geometry

Question No (12)

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Question No (12)

The diagram shows a quadrilateral ABCD in which the point A is $(-1, -1)$, the point B is $(3, 6)$ and the point C is $(9, 4)$. The diagonals AC and BD intersect at M. Angle BMA = 90° and $BM = MD$. Calculate

- (i) the coordinates of M and D,
- (ii) the ratio AM: MC.

Solution

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(i) Given point
 $A(-1, -1)$, $C(9, 4)$

$$\begin{aligned}\text{gradient of AC} &= \frac{4 - (-1)}{9 - (-1)} \\ &= \frac{4+1}{9+1} = \frac{5}{10} \\ &= \frac{1}{2}\end{aligned}$$

Equation of line AC passing through $A(-1, -1)$
and gradient $\frac{1}{2}$

$$y - y_1 = \frac{1}{2} (x - x_1)$$

$$y - (-1) = \frac{1}{2} (x - (-1))$$

$$y + 1 = \frac{1}{2} (x + 1)$$

$$2(y + 1) = x + 1$$

$$2y + 2 = x + 1$$

$$2y = x + 1 - 2$$

$$2y = x - 1 \rightarrow \textcircled{1}$$

As BD is perpendicular to AC, then gradient of BD will be

$$BD = -2$$

Equation of BD passing through B(3,6) and gradient -2 is,

$$y - y_1 = -2(x - x_1)$$

$$y - 6 = -2(x - 3)$$

$$y - 6 = -2x + 6$$

$$y = -2x + 6 + 6$$

$$y = -2x + 12 \rightarrow (2)$$

To find point M, we shall solve equation ① & ② simultaneously
put equation ② in ①

$$2(-2x + 12) = x - 1$$

$$-4x + 24 = x - 1$$

$$-4x - x = -1 - 24$$

$$-5x = -25$$

$$x = 5$$

substitute $x=5$ in (2)

$$y = -2(5) + 12$$

$$= -10 + 12$$

$$y = 2$$

so coordinates of M (5, 2)

let D has coordinates (x, y)

$$\text{mid-point of BD} = \left(\frac{x+3}{2}, \frac{y+6}{2} \right)$$

$$(5, 2) = \left(\frac{x+3}{2}, \frac{y+6}{2} \right) \quad \because \text{mid-point of BD is (5, 2)}$$

By comparison

$$\frac{x+3}{2} = 5$$

$$\frac{y+6}{2} = 2$$

$$x+3 = 10$$

$$y+6 = 4$$

$$x = 10 - 3$$

$$y = 4 - 6$$

$$x = 7$$

$$y = -2$$

$$\therefore D(7, -2)$$

(ii)

$$A(-1, -1), M(5, 2)$$

$$\begin{aligned}
 |AM| &= \sqrt{(5-(-1))^2 + (2-(-1))^2} \\
 &= \sqrt{(5+1)^2 + (2+1)^2} \\
 &= \sqrt{6^2 + 3^2} \\
 &= \sqrt{36+9} \\
 &= \sqrt{45} \\
 &= \sqrt{3 \times 3 \times 5}
 \end{aligned}$$

$$|AM| = 3\sqrt{5}$$

now $m(5, 2)$, $c(9, 4)$

$$\begin{aligned}
 |MC| &= \sqrt{(9-5)^2 + (4-2)^2} \\
 &= \sqrt{4^2 + 2^2} \\
 &= \sqrt{16+4} \\
 &= \sqrt{20} \\
 &= \sqrt{2 \times 2 \times 5} \\
 &= 2\sqrt{5}
 \end{aligned}$$

Hence $AM : MC$
 $3\sqrt{5} : 2\sqrt{5}$
 $= 3 : 2$

