

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

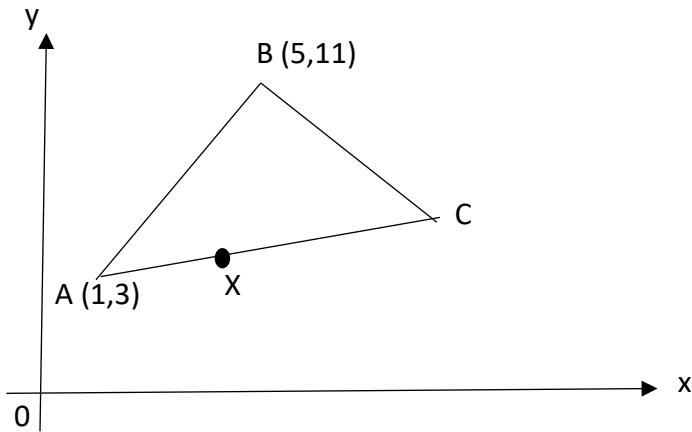
Topic 3-Coordinate Geometry

Question No (14)

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Question No (14)

The diagram shows a triangle ABC in which A has coordinates (1,3), B has coordinates (5,11) and angle ABC is 90° . The point X (4,4) lies on AC. Find

- (i) the equation of BC,
- (ii) the coordinates of C.

Solution

On Next page

②

A(1,3) and B(5,11)

How to find gradient
from two points

A(x₁, y₁), B(x₂, y₂)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient of AB} = \frac{11-3}{5-1}$$

$$= \frac{8}{4}$$

$$= 2$$

if m₁ is the gradient of one line and m₂ is the gradient of second line
if these are perpendicular to each other then
(m₁)(m₂) = -1

As BC is perpendicular to AB,
then gradient of BC is $= -\frac{1}{2}$

Equation of line from one
point P(x₁, y₁) and gradient m
is

$$y - y_1 = m(x - x_1)$$

Equation of BC, passing through B(5,11)
and gradient $-\frac{1}{2}$ is

$$y - 11 = -\frac{1}{2}(x - 5)$$

$$2(y - 11) = -(x - 5)$$

$$2y - 22 = -x + 5$$

$$x + 2y = 5 + 22$$

$$x + 2y = 27 \rightarrow \textcircled{1}$$

(ii) Now we find the point C
from equation BC

$$x + 2y = 27$$

$$x = 27 - 2y \rightarrow (1)$$

As AX is parallel to AC, so

$$\begin{aligned} \text{gradient of AX} &= \frac{4-3}{4-1} \\ &= \frac{1}{3} \end{aligned}$$

Equation of AC passing through A (1, 3)
and gradient $\frac{1}{3}$

$$y - y_1 = \frac{1}{3}(x - x_1)$$

$$y - 3 = \frac{1}{3}(x - 1)$$

$$3(y - 3) = x - 1$$

$$3y - 9 = x - 1$$

$$3y = x - 1 + 9$$

$$3y = x + 8 \rightarrow (2)$$

solving (1) & (2)

$$3y = 27 - 2y + 8$$

$$3y + 2y = 35 \Rightarrow y = \frac{35}{5} = 7$$

put $y=7$ in ①

$$x = 27 - 2(7)$$

$$= 27 - 14$$

$$x = 13$$

$\therefore$ coordinates of $C(13, 7)$.