Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 3-Coordinate Geometry

Question No (15)

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The point R is the reflection of the point (−1, 3) in the line 3y + 2x = 33. Find by calculation the coordinates of R.

Solution

Given line

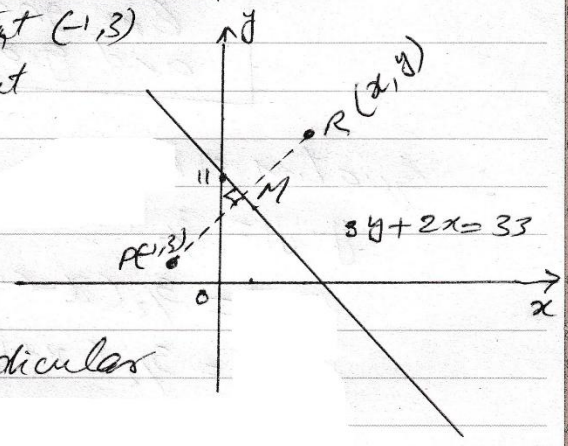
$3y+2x = 33 \rightarrow (1)$

x	0	3
y	11	9

Let P be the point (−1, 3) and M is the point on the given line such that

$|PM| = |MR|$

and PM is perpendicular to the line.



Given line

$3y + 2x = 33$

$3y = -2x + 33$

$y = -\frac{2}{3}x + \frac{33}{3}$

$y = -\frac{2}{3}x + 11 \rightarrow (1)$

comparing with Equation of line in x-intercept form

$y = mx + c$

$\Rightarrow m = -\frac{2}{3}$

if m1 is the gradient of one line and m2 is the gradient of second line if these are perpendicular to each other then
(m1)(m2)= -1

gradient of $PM = 3/2$

Equation of line from one point $P(x_1, y_1)$ and gradient m is

$$y - y_1 = m(x - x_1)$$

∴ Equation of PM is

$$y - y_1 = \frac{3}{2}(x - x_1)$$

$$y - 3 = \frac{3}{2}(x - (-1))$$

$$y - 3 = \frac{3}{2}(x + 1)$$

$$2(y - 3) = 3(x + 1)$$

$$2y - 6 = 3x + 3$$

$$2y = 3x + 3 + 6$$

$$2y = 3x + 9$$

$$y = \frac{3}{2}x + \frac{9}{2} \longrightarrow (2)$$

Solving Equation (1) & (2)

$$-\frac{2}{3}x + 11 = \frac{3}{2}x + \frac{9}{2}$$

$$-\frac{2}{3}x - \frac{3}{2}x = \frac{9}{2} - 11$$

$$\frac{-4x - 9x}{6} = \frac{9 - 22}{2}$$

$$\frac{-13x}{6} = \frac{-13}{2}$$

$$-\frac{x}{6} = -\frac{1}{2}$$

$$x = 6/2$$

$$x = 3$$

put $x = 3$ in (1)

$$y = -\frac{2}{3}(3) + 11$$

$$= -2 + 11$$

$$y = 9$$

∴ coordinates of M are $(3, 9)$

As M is the mid-point of PR

$$\left(\frac{x-1}{2}, \frac{y+3}{2}\right) = (3, 9)$$

By comparison

$$\frac{x-1}{2} = 3, \qquad \frac{y+3}{2} = 9$$

$$x - 1 = 6 \qquad y + 3 = 9 \times 2$$

$$x = 6 + 1 \qquad y = 18 - 3$$

$$x = 7 \qquad y = 15$$

∴ $R(7, 15)$