Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 3-Coordinate Geometry

Question No (17)

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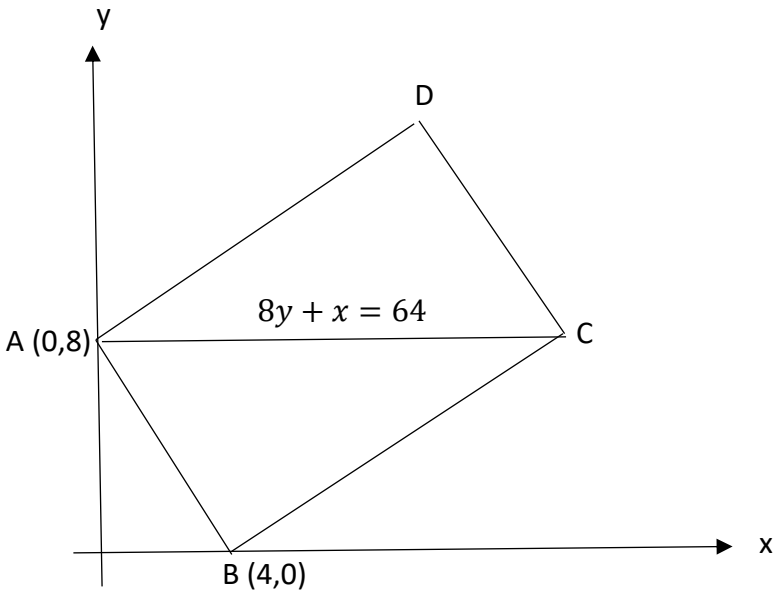

The diagram shows a rectangle ABCD in which point A is (0, 8) and point B is (4, 0). The diagonal AC has equation $8y + x = 64$. Find, by calculation, the coordinates of C and D.

Solution

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Given $A(0,2)$ and $B(4,0)$

How to find gradient from two points $A(x_1, y_1)$, $B(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Gradient of $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0-2}{4-0}$

$= -2$

if m1 is the gradient of one line and m2 is the gradient of second line if these are perpendicular to each other then
(m1)(m2)= -1

As AB is perpendicular to B ($AB \perp BC$)

gradient of $BC = \frac{1}{2}$

Equation of line from one point $P(x_1, y_1)$ and gradient m is

$$y - y_1 = m(x - x_1)$$

Equation of BC passing through $B(4,0)$ and gradient $\frac{1}{2}$

$$y - y_1 = \frac{1}{2}(x - x_1)$$

$$y - 0 = \frac{1}{2}(x - 4)$$

$2(y-0)=1(x-4)$

$2y-0=x-4$

$2y-x=-4 \rightarrow (1)$

Given equation of AC

$8y+x=64 \rightarrow (2)$

solving (1) & (2)

$2y-x=-4$

$8y+x=64$

$10y=60$

$y=\frac{60}{10}$

$y=6$

put $y=6$ in (1)

$2(6)-x=-4$

$12-x=-4$

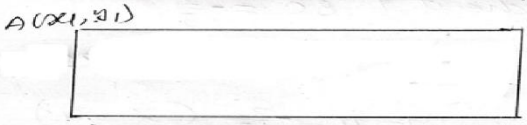
$x=12+4$

$x=16$

∴ coordinates of C are (16,6)

Now we find the coordinates of D(x,y)

Property of Rectangle



$x_1 + x_3 = x_2 + x_4$

and $y_1 + y_3 = y_2 + y_4$

$x + 4 = 0 + 16$, $y + 0 = 8 + 6$

$x = 16 - 4$ $\quad y = 14$

$x = 12$

∴ coordinates of D are (12, 14)