Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 3-Coordinate Geometry

Question No (19)

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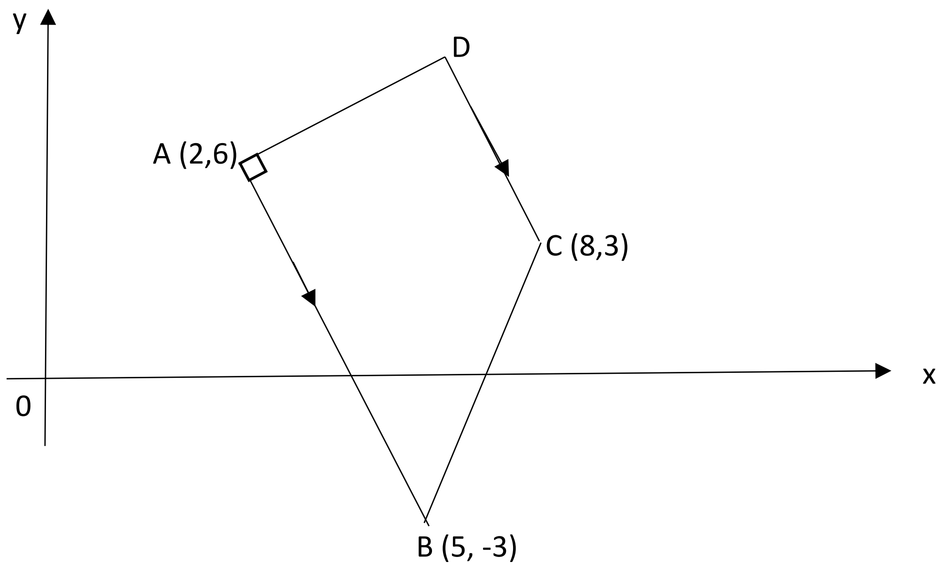

The diagram shows a trapezium ABCD in which AB is parallel to DC and angle BAD is 90^0 . The coordinates of A, B and C are (2, 6),(5, −3) and (8, 3) respectively.

(i) Find the equation of AD.

(ii) Find, by calculation, the coordinates of D.

The point E is such that ABCE is a parallelogram.

(iii) Find the length of BE.

<u>Solution</u>

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From the diagram

(i) $A(2,6)$ and $B(5,-3)$

How to find gradient from two points
$A(x_1, y_1)$, $B(x_2, y_2)$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient of } AB = \frac{-3-6}{5-2}$$
$$= \frac{-9}{3}$$
$$= -3$$

if m1 is the gradient of one line and m2 is the gradient of second line if these are perpendicular to each other then (m1)(m2)= -1

As $AB \perp AD$

$\Rightarrow$ gradient of $AD = \frac{1}{3}$

Equation of line from one point $P(x_1, y_1)$ and gradient m is
$$y - y_1 = m(x - x_1)$$

Equation of line AD passing through $A(2,6)$ and gradient $\frac{1}{3}$
$$y - 6 = \frac{1}{3}(x-2)$$

$$3(y-6)=x-2$$
$$3y-18=x-2$$
$$3y=x-2+18$$
$$3y=x+16 \rightarrow (1)$$

(ii)

if m1 is the gradient of one line and m2 is the gradient to second line ,if these lines are parallel then
m1=m2

Given AB is parallel to DC

$\therefore$ gradient of AB = gradient of DC

$\therefore$ gradient of DC $=-3$

Equation of line from one point $P(x_1, y_1)$ and gradient m is

$$y-y_1=m(x-x_1)$$

Equation of DC passing Through C(8,3) and gradient -3

$$y-3=-3(x-8)$$
$$y-3=-3x+24$$
$$y=-3x+24+3$$
$$y=-3x+27 \rightarrow (2)$$

solving equation ① & ② simultaneously

put value of ② in ①

$3(-3x+27) = x+16$

$-9x + 81 = x+16$

$81-16 = x+9x$

$10x = 65$

$x = \frac{65}{10}$

$x = 6.5$

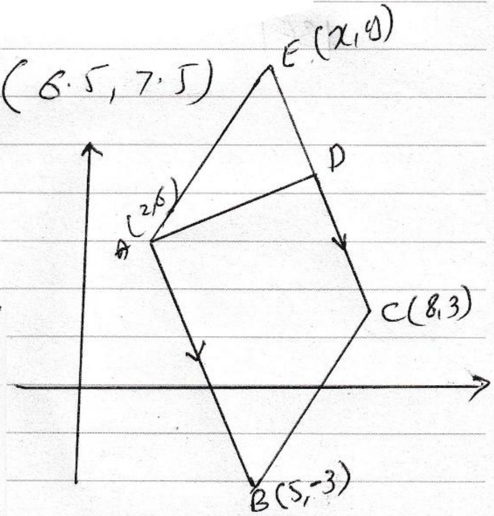
put $x = 6.5$ in ②

$y = -3(6.5) + 27$

$= -19.5 + 27$

$y = 7.5$

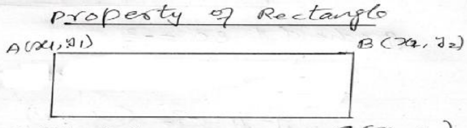
$\therefore$ coordinates of $D\,(6.5, 7.5)$



(iii)

let coordinates of E be (x, y)

Property of Rectangle



$x_1 + x_3 = x_2 + x_4$

and $y_1 + y_3 = y_2 + y_4$

$x+5=2+8 \quad , \quad y-3=6+3$

$x+5=10 \qquad y-3=9$

$x=10-5 \qquad y=9+3$

$x=5 \qquad y=12$

$\therefore$ coordinates of $E(5,12)$

$$\boxed{\begin{array}{l}\text{Distance between two points } P(x_1,y_1),\ Q(x_2,y_2) \\ |PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\end{array}}$$

$$|BE| = \sqrt{(5-5)^2+(12-(-3))^2}$$

$$= \sqrt{(0)^2+(12+3)^2}$$

$$= \sqrt{(15)^2}$$

$|BE| = 15$ units