

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 3-Coordinate Geometry

Question No (21)

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Question No (21)

Points A, B and C have coordinates A (-3,7), B (5,1) and C (-1, k), where k is a constant.

- (i) Given that $AB = BC$, calculate the possible values of k.

The perpendicular bisector of AB intersects the x-axis at D.

- (ii) Calculate the coordinates of D.

Solution

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Given points

$$A(-3, 7), B(5, 1), C(-1, k)$$

(i)

Formula for distance
between two points $A(x_1, y_1)$,
 $B(x_2, y_2)$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given condition

$$|AB| = |BC|$$

$$\sqrt{(5 - (-3))^2 + (1 - 7)^2} = \sqrt{(-1 - 5)^2 + (k - 1)^2}$$

$$\sqrt{(8)^2 + (-6)^2} = \sqrt{(-6)^2 + (k - 1)^2}$$

$$\sqrt{64 + 36} = \sqrt{36 + (k - 1)^2}$$

$$\sqrt{100} = \sqrt{36 + (k - 1)^2}$$

Squaring both sides

$$100 = 36 + (k - 1)^2$$

$$100 - 36 = (k - 1)^2$$

$$64 = (k-1)^2$$

$$(k-1)^2 = 64$$

$$k-1 = \pm 8$$

$$k-1 = 8$$

$$k = 8+1$$

$$k = 9$$

$$k-1 = -8$$

$$k = -8+1$$

$$k = -7$$

possible values of k are 9, -7

(ii)

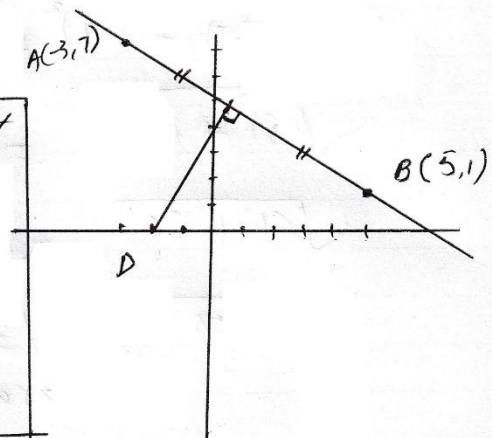
$A(-3, 7), B(5, 1)$

Formula for gradient
from two points

$A(x_1, y_1), B(x_2, y_2)$

$$\text{Gradient of } AB = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient of } AB = \frac{1-7}{5+3}$$



$$\begin{aligned}\text{gradient of } AB &= \frac{-6}{8} \\ &= -3/4\end{aligned}$$

Formula gradient of perpendicular bisector

negative reciprocal of gradient of
i.e. $-1/(-3/4) = 4/3$

$\Rightarrow$ gradient of perpendicular bisector of
AB is $4/3$

Formula for mid-point of two
points $A(x_1, y_1), B(x_2, y_2)$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{mid-point of } AB = \left(\frac{-3+5}{2}, \frac{7+1}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{8}{2} \right)$$

$$M = (1, 4)$$

Formula for equation of line from one point $P(x_1, y_1)$ and gradient m is

$$y - y_1 = m(x - x_1)$$

Equation of perpendicular bisector AB through $M(1, 4)$

$$y - y_1 = m(x - x_1)$$

$$3(y - 4) = 4(x - 1)$$

$$3y - 12 = 4x - 4$$

$$3y - 4x = 12 - 4$$

$$3y - 4x = 8 \rightarrow \textcircled{1}$$

As this line cut the x -axis, $\Rightarrow y = 0$

So Equation (1) becomes

$$3(0) - 4x = 8$$

$$-4x = 8$$

$$x = -2$$

$$x = -2$$

$\therefore$ coordinates of D $(-2, 0)$