Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 3-Coordinate Geometry

Question No (23)

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The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, intersects the x- and y-axes at the points A and B respectively. The mid-point of AB lies on the line $2x + y = 10$ and the distance AB = 10. Find the values of a and b.

Solution

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Given equation of line

$$\frac{x}{a} + \frac{y}{b} = 1 \longrightarrow (1)$$

For x-intercept, put $y = 0$ in (1)

$$\frac{x}{a} = 1$$

$$x = a$$

$\therefore A(a, 0)$

For y-intercept, put $x = 0$ in (1)

$$\frac{y}{b} = 1$$

$$y = b$$

$\therefore B(0, b)$

> Mid-point formula for two points $A(x_1, y_1), B(x_2, y_2)$
>
> $$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Mid-point of $AB = \left(\frac{a+0}{2}, \frac{0+b}{2}\right)$

$$= \left(a/2, b/2\right)$$

As this mid point lie on given line

$$2x + y = 10$$

$\Rightarrow 2(a/2) + b/2 = 10$

$a + b/2 = 10$

$2a + b = 20 \longrightarrow (2)$

Distance formula between two points

$A(x_1, y_1)$, $B(x_2, y_2)$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

As given

$|AB| = 10$

$\sqrt{(0-a)^2 + (b-0)^2} = 10$

$\sqrt{a^2 + b^2} = 10$

squaring both sides

$a^2 + b^2 = 100 \longrightarrow (3)$

solving equation (2) and (3)

from (2)

$2a + b = 20$

$b = 20 - 2a$

put in equation (3)

$$a^2 + (20-2a)^2 = 100$$

$$a^2 + (20)^2 - 2(20)(2a) + (2a)^2 = 100$$

$$a^2 + 400 - 80a + 4a^2 = 100$$

$$5a^2 - 80a + 400 - 100 = 0$$

$$5a^2 - 80a + 300 = 0$$

$$5(a^2 - 16a + 60) = 0$$

$$\Rightarrow a^2 - 16a + 60 = 0$$

By factorization

$$a^2 - 10a - 6a + 60 = 0$$

$$a(a-10) - 6(a-10) = 0$$

$$(a-10)(a-6) = 0$$

$$a-10 = 0, \quad a-6 = 0$$

$$a = 10, \quad a = 6$$

put $a = 10$ in eqn (2)

$$2(10) + b = 20$$

$$20 + b = 20$$

$$b = 20 - 20$$

$$b = 0$$

ignore as a, b are +ve.

put $a = 6$ in (2)

$$2(6) + b = 20$$

$$b = 20 - 12$$

$$b = 8$$

$\therefore a = 6, \; b = 8$