Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 3-Coordinate Geometry

Question No (29)

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The equation of a circle with center C is $x^2 + y^2 - 8x + 4y - 5 = 0.$

(a) Find the radius of the circle and the coordinates of C.

The point P (1,2) lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5.$

The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q.

The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T.

Solution

On the next page

Equation of circle

$$x^2+y^2-8x+4y-5=0 \rightarrow (1)$$

(a)

$$\boxed{\begin{array}{l} \text{General equation of circle} \\ x^2+y^2+2gx+2fy+c=0 \rightarrow (2) \\ \text{where,} \\ \text{centre} = (-g, -f) \\ \text{Radius} = \sqrt{g^2+f^2-c} \end{array}}$$

compare equation (1) & (2)

$2g = -8$, $2f = 4$

$g = -4$, $f = 2$

$\therefore$ centre is, $C(-g, -f) = C(4, -2)$

radius, $r = \sqrt{g^2+f^2-c}$

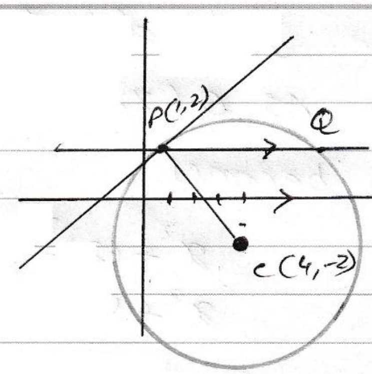
$= \sqrt{(-4)^2+(2)^2-(-5)}$

$= \sqrt{16+4+5}$

$= \sqrt{25}$

$r = 5$

(b) $C(4,-2)$, $P(1,2)$



Gradient of $CP = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{2-(-2)}{1-4}$

$= \dfrac{2+2}{-3}$

Gradient of $CP = -\frac{4}{3}$

As CP is perpendicular to tangent, so gradient of tangent is, $\frac{3}{4}$ (negative reciprocal)

$\therefore$ Equation of tangent

$y - y_1 = \frac{3}{4}(x - x_1)$

$y - 2 = \frac{3}{4}(x-1)$ $\quad \because P(1,2)$

$4(y-2) = 3(x-1)$

$4y - 8 = 3x - 3$

$4y = 3x - 3 + 8$

$4y = 3x + 5$

(c) As PQ is parallel to x-axis, so y-coordinate of Q is 2 i.e. $Q(x, 2)$

As $Q(x, 2)$ lie on circle, so Equation (1) becomes

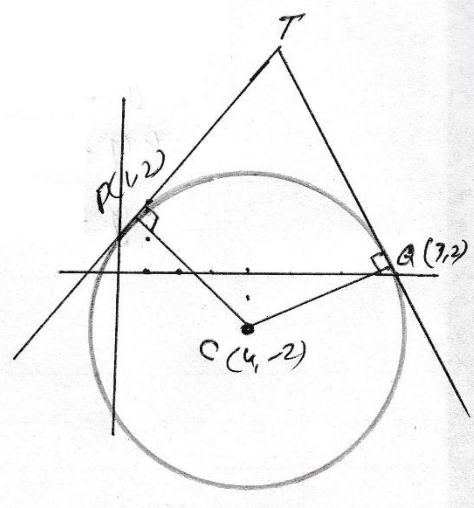
$$x^2 + (2)^2 - 8x + 4(2) - 5 = 0$$
$$x^2 + 4 - 8x + 8 - 5 = 0$$
$$x^2 - 8x + 7 = 0$$

By Factorization

$$x^2 - 7x - x + 7 = 0$$
$$x(x-7) - 1(x-7) = 0$$
$$(x-7)(x-1) = 0$$
$$x - 7 = 0, \quad x - 1 = 0$$
$$x = 7, \quad x = 1$$

$\Rightarrow x = 7$

$\therefore Q(7, 2)$



(d)

$C(4, -2)$, $Q(7, 2)$

gradient of $CQ = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{2 - (-2)}{7 - 4}$$

Gradient of CQ $= 4/3$

AS CQ is perpendicular to TQ, so

Gradient of QT $= -3/4$ (−ve reciprocal of CQ gradient)

Equation of QT passing through Q(7, 2)

$y - y_1 = -3/4(x - x_1)$

$y - 2 = -3/4(x - 7)$

$4(y - 2) = -3(x - 7)$

$4y - 8 = -3x + 21$

$4y = -3x + 21 + 8$

$4y = -3x + 29 \longrightarrow ①$

From part (b) equation of tangent is

$4y = 3x + 5 \longrightarrow ②$

solving ① & ②

$-3x + 29 = 3x + 5$

$29 - 5 = 3x + 3x$

$24 = 6x$

$x = 4$

put $x=4$ in equation (1)

$$4y = -3(4) + 29$$

$$= -12 + 29$$

$$4y = 17$$

$$y = \frac{17}{4}$$

$\therefore$ coordinates of T ~~ase~~ $\left(4, \frac{17}{4}\right)$