

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 3-Coordinate Geometry

Question No (30)

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Question No (30)

A circle has center at the point B (5,1). The point A (−1, −2) lies on the circle.

(a) Find the equation of the circle.

Point C is such that AC is a diameter of the circle. Point D has coordinates (5,16).

(b) Show that DC is a tangent to the circle.

The other tangent from D to the circle touches the circle at E.

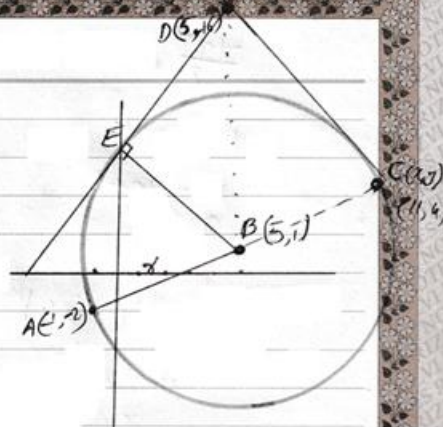
(c) Find the coordinates of E.

Solution

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(a)
 $A(-1, -2), B(5, 1)$
 radius
 $r = |AB|$

$$\begin{aligned}
 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(5 - (-1))^2 + (1 - (-2))^2} \\
 &= \sqrt{6^2 + 3^2} \\
 &= \sqrt{36 + 9} \\
 &r = \sqrt{45}
 \end{aligned}$$



$\therefore$ equation of circle is

$$(x - 5)^2 + (y - 1)^2 = (\sqrt{45})^2$$

$$x^2 - 10x + 25 + y^2 - 2y + 1 = 45$$

$$x^2 + y^2 - 10x - 2y + 25 + 1 - 45 = 0$$

$$x^2 + y^2 - 10x - 2y - 19 = 0$$

(b) let $C(x, y)$ be the point such that AC is passing through centre $B(5, 1)$.
 As $B(5, 1)$ is the mid-point of AC, so

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = (5, 1)$$

$$\left(\frac{-1 + x}{2}, \frac{-2 + y}{2} \right) = (5, 1)$$

comparing

$$\frac{-1+x}{2} = 5$$

$$-1+x=10$$

$$x=10+1$$

$$x=11$$

$$\frac{-2+y}{2} = 1$$

$$-2+y=2$$

$$y=2+2$$

$$y=4$$

$$\therefore C(11, 4)$$

$$\text{Gradient of AC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - (-2)}{11 - (-1)}$$

$$= \frac{4+2}{11+1} = \frac{6}{12}$$

$$= \frac{1}{2}$$

As AC is perpendicular to CD, so
gradient of CD is,

$$-2$$

(-ve reciprocal of
gradient of AC)

As

$$(\text{Gradient of AC}) \times (\text{Gradient of CD})$$

$$\left(\frac{1}{2}\right)(-2) = -1$$

So DC is tangent to the circle at C.

(c) From the figure in part (b),
BCDE is a kite.

As x-coordinates of B and D are
same, it shows that point E is the
reflection of point C in BD

$\therefore$ coordinates of E $(-1, 4)$