

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

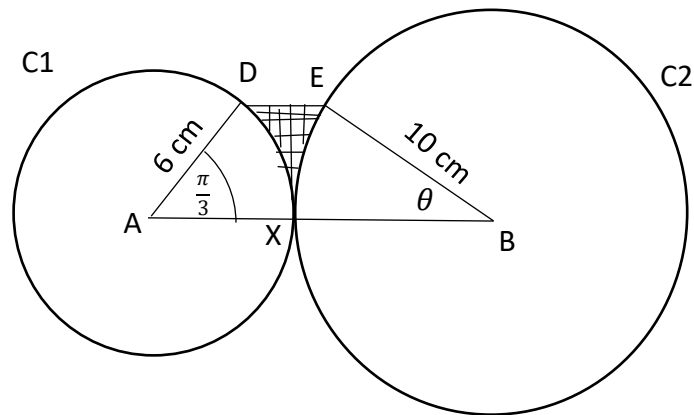
Topic 4-Circular Measure

Question No (9)

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Question No (9)

The diagram shows a circle C_1 touching a circle C_2 at a point X . Circle C_1 has center A and radius 6 cm, and circle C_2 has center B and radius 10 cm. Points D and E lie on C_1 and C_2 respectively and DE is parallel to AB . Angle $DAX = \frac{\pi}{3}$ radians and angle $EBX = \theta$ radians.

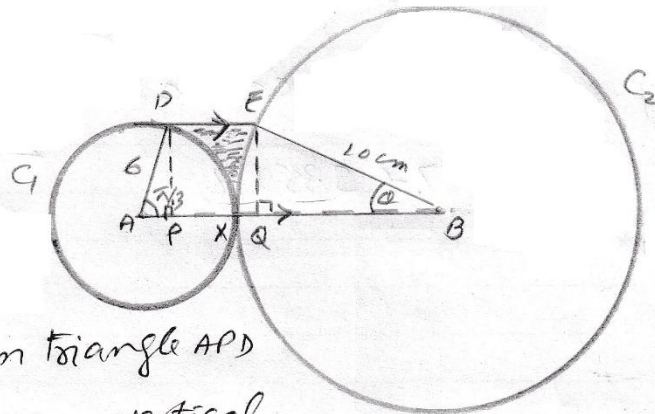
(i) By considering the perpendicular distances of D and E from AB , show that the exact value of θ is $\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$.

(ii) Find the perimeter of the shaded region, correct to 4 significant figures

Solution

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(i)



from triangle APD

$$\sin 73 = \frac{\text{vertical}}{\text{hypotenuse}}$$

$$= \frac{DP}{AD}$$

$$\sin 73 = \frac{DP}{6}$$

$$DP = 6 \sin 73$$

$$= 6 \left(\frac{\sqrt{3}}{2} \right)$$

$$DP = 3\sqrt{3}$$

now from triangle EBQ

$$\sin \alpha = \frac{EQ}{EB}$$

$$= \frac{EQ}{10}$$

$$EQ = 10 \sin \alpha$$

$$\sin \alpha \quad DE \parallel AB$$

($\because$ DE is parallel to AB)

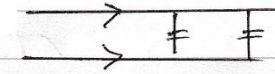
$$DP = EQ$$

$$3\sqrt{3} = 10 \sin \theta$$

$$\sin \theta = \frac{3\sqrt{3}}{10}$$

$$\theta = \sin^{-1}\left(\frac{3\sqrt{3}}{10}\right) \text{ proved.}$$

$$\therefore DP = 3\sqrt{3} \text{ and } EQ = 10 \sin \theta$$



(ii) Now we find out the perimeter of the shaded region
As from part (i)

$$\theta = \sin^{-1}\left(\frac{3\sqrt{3}}{10}\right) = 0.546 \text{ radians.}$$

$$\text{Arc length } \widehat{DX} = r\theta$$

$$= S = r\theta$$

$$= 6\left(\frac{\pi}{3}\right)$$

$$= 2\pi \text{ cm}$$

$$\text{Arc length of } \widehat{EX} = r\theta$$

$$= S = r\theta$$

$$= 10(0.546)$$

$$\therefore \theta = 0.546$$

$$= 5.46 \text{ cm}$$

From triangle APD

$$\cos\left(\frac{\pi}{3}\right) = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{AP}{6}$$

$$AP = 6 \cos(\theta_B)$$

$$= 6 \left(\frac{1}{2} \right)$$

$$AP = 3 \text{ cm}$$

From $\triangle EQB$

$$\cos(0.546) = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\therefore \theta = 0.546$$

$$\cos(0.546) = \frac{QB}{10}$$

$$QB = 10 \cos(0.546)$$

$$QB = 8.546 \text{ cm}$$

From Fig

$$AB = AX + XB$$

$$= 6 + 10$$

$$AB = 16 \text{ cm}$$

radii of circles

From Fig

$$AB = AP + PQ + QB$$

$$AB = AP + DE + QB \quad \because PQ = DE$$

$$DE = AB - AP - QB$$

$$= 16 - 3 - 8.546$$

$$DE = 4.454 \text{ cm}$$

Now Perimeter of shaded region

$$= DE + \widehat{EX} + \widehat{DX}$$

$$= 4.454 + 5.46 + 2\pi$$

$$= 16.20 \text{ cm}$$

