

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

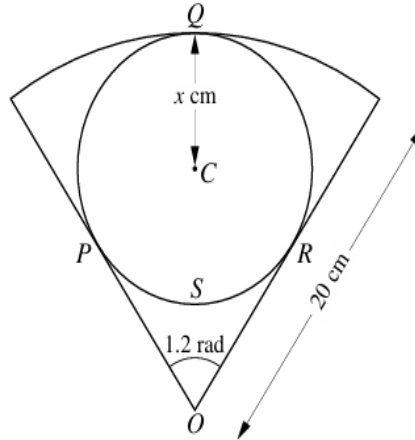
Topic 4-Circular Measure

Question No (11)

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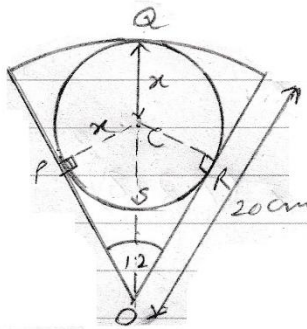
The diagram shows a sector of a circle with center O and radius 20 cm. A circle with center C and radius x cm lies within the sector and touches it at P , Q and R . Angle $POR = 1.2$ radians.

- (i) Show that $x = 7.218$, correct to 3 decimal places.
- (ii) Find the total area of the three parts of the sector lying outside the circle with center C .
- (iii) Find the perimeter of the region $OPSR$ bounded by the arc PSR and the lines OP and OR .

Solution

On Next page

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FROM $\triangle CPO$

$\angle CPO = 90^\circ$ $\because$ OP is tangent to the circle.

$\cos \angle CPO = \frac{CP}{OP} = 0.6$ $\because$ OQ bisects the angle POR

$\sin \angle CPO = \frac{\text{vertical}}{\text{hypotenuse}}$

$$\sin 0.6 = \frac{x}{20-x}$$

$$\begin{aligned} \because OC &= OQ - CQ \\ x &= 20 - x \end{aligned}$$

$$20 - x = \frac{x}{\sin 0.6}$$

$$20 = \frac{x}{\sin 0.6} + x$$

$$20 = \left(\frac{1}{\sin 0.6} + 1 \right) x$$

$$x = \frac{20}{\frac{1}{\sin 0.6} + 1} = \frac{20}{2.771} = 7.217$$

(i) Now we find the area outside of the circle.

Total area outside the circle

= Area of the sector - area of the circle

$$= \frac{1}{2} r^2 \theta - \pi x^2$$

$$= \frac{1}{2} (20)^2 (1.2) - \pi (7.218)^2 \quad \begin{matrix} x = 7.218 \\ \theta = 1.2 \end{matrix}$$

$$= 240 - 163.675$$

$$= 76.325 \text{ cm}^2$$

(ii) Now we find perimeter of the region PSR and the lines OP and OR from $\triangle OPC$

$$\tan 0.6 = \frac{\text{vertical}}{\text{Base}}$$

$$= \frac{PC}{OP}$$

$$\tan 0.6 = \frac{x}{OP}$$

$$OP = \frac{x}{\tan 0.6}$$

$$= \frac{7.218}{\tan 0.6}$$

$$OP = 10.55 \text{ cm}$$

$$\Rightarrow OP = OR \quad \because \triangle OPC \text{ and } \triangle ORC \text{ are congruent.}$$

$$\Rightarrow OR = 10.55 \text{ cm}$$

$$\angle PCR + 1.2 = \pi \quad (\because \angle PCR \text{ and } \angle ORC \text{ are supplement angle due to quadrilateral})$$

$$\angle PCR = \pi - 1.2$$

$$\angle PCR = 1.942$$

now arc length of $\widehat{PSR}$ $\because s = r\theta$

$$= r \angle PCR$$

$$= (7.218)(1.942)$$

$$= 14.02$$

$$\therefore \text{perimeter of } OPSR = OP + \widehat{PSR} + OR$$

$$= 10.55 + 14.02 + 10.55$$

$$= 35.1 \text{ cm}$$