Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 4-Circular Measure

Question No (12)

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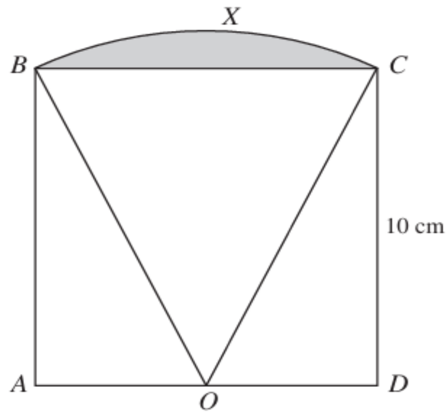

The diagram shows a square $ABCD$ of side 10 cm. The mid-point of AD is O and BXC is an arc of a circle with centre O.

(i) Show that angle BOC is 0.9273 radians, correct to 4 decimal places.

(ii) Find the perimeter of the shaded region.

(iii) Find the area of the shaded region.

Solution

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(i)

In triangle AOB

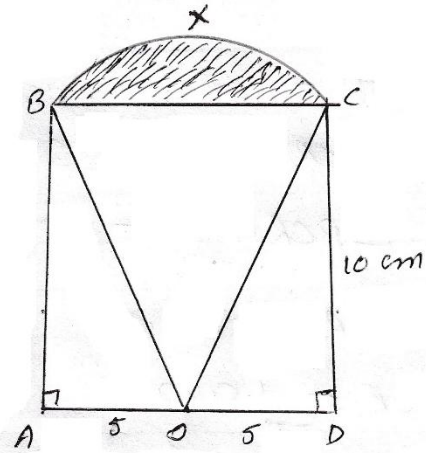
$\tan A\hat{O}B = \frac{\text{vertical}}{\text{Base}}$

$= \frac{10}{5}$

$= 2$

$A\hat{O}B = \tan^{-1}(2)$

$A\hat{O}B = 1.1071$



From $\triangle$ COD

$\tan C\hat{O}D = \frac{\text{vertical}}{\text{Base}}$

$= \frac{10}{5}$

$= 2$

$C\hat{O}D = \tan^{-1}(2)$

$C\hat{O}D = 1.1071$

Base = 5 ∵ O is mid-point of AD.

$\therefore \quad \pi = A\hat{O}B + B\hat{O}C + C\hat{O}D$

$\pi = 1.1071 + B\hat{O}C + 1.1071$

$B\hat{O}C = \pi - 1.1071 - 1.1071$

$B\hat{O}C = 0.9273$

(ii) Now we shall find out the perimeter of shaded region.

From fig radius of the sector is

In $\triangle$ AOB, use Pythagoras Theorem

$$\text{hypotenuse} = \sqrt{(\text{vertical})^2 + (\text{base})^2}$$

$$= \sqrt{(10)^2 + (5)^2}$$

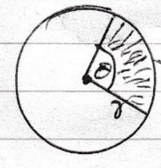
$$= \sqrt{100 + 25}$$

$$OB = \sqrt{125} \text{ cm}$$

$$=$$

Arc length Formula

$$s = r\theta$$



$$\text{Arc length } \widehat{BXC} = r\theta$$

$$= OB \; B\hat{O}C$$

$$= \sqrt{125}\;(0.9273)$$

$$= 10.368$$

$\therefore$ perimeter of shaded region

$$= BC + \widehat{BXC}$$

$$= 10 + 10.368$$

$$= 20.4 \text{ cm}$$

$BC = 10$
$\because$ ABCD is square of side 10

(iii) Now we shall find the area of the shaded region.

Area of shaded region = Area of sector OBC − area of triangle BOC

$= \frac{1}{2} r^2 \theta - \frac{1}{2}(OB)(OC)\sin(B\hat{O}C)$

$= \frac{1}{2}(\sqrt{125})^2(0.9273) - \frac{1}{2}(\sqrt{125})(\sqrt{125})\sin 0.9273$

$= \frac{1}{2}(125)(0.9273) - \frac{1}{2}(125)(\sin(0.9273))$

$= 57.9562 - 50.000$

$= 7.95\ cm^2$