

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

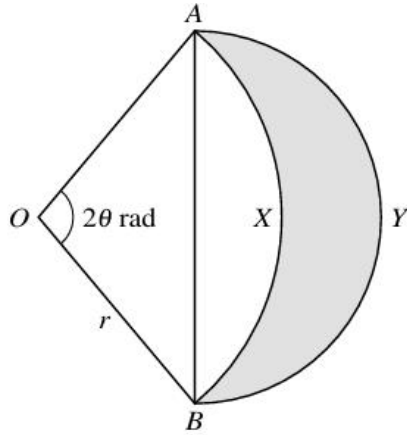
Topic 4-Circular Measure

Question No (16)

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Question No (16)

In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius r . Angle $AOB = 2\theta$ radians. Find an expression, in terms of r and θ , for the area of the shaded region.

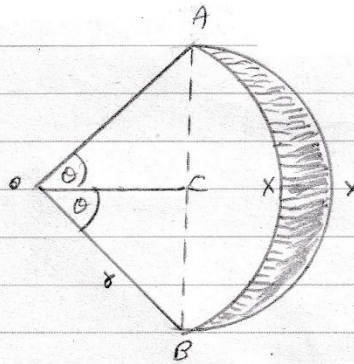
Solution

On Next page

From the $\triangle OBC$

$$\sin \theta = \frac{\text{vertical}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{BC}{r}$$



$$BC = r \sin \theta$$

$$\therefore \text{radius of semi circle } AXB = r \sin \theta$$

$$\text{Area of semi circle} = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} (\pi) (BC)^2$$

$$= \frac{1}{2} \pi r^2 \sin^2 \theta$$

$$\text{Area of segment } AXB = \text{area of sector} - \text{area of triangle } OAB$$

$$= \frac{1}{2} r^2 (2\theta) - \frac{1}{2} OA \cdot OB \sin 2\theta$$

$$= \frac{1}{2} r^2 (2\theta - \sin 2\theta)$$

$$= \frac{1}{2} r^2 (2\theta) - \frac{1}{2} r^2 \sin 2\theta$$

$$\therefore \text{Area of shaded region}$$

$$= \text{Area of semi circle} - \text{area of segment } AXB$$

$$= \frac{1}{2} \pi r^2 \sin^2 \alpha - \frac{1}{2} r^2 (2\alpha - \sin 2\alpha)$$

$$= \frac{1}{2} r^2 (\pi \sin^2 \alpha - 2\alpha + \sin 2\alpha)$$