

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

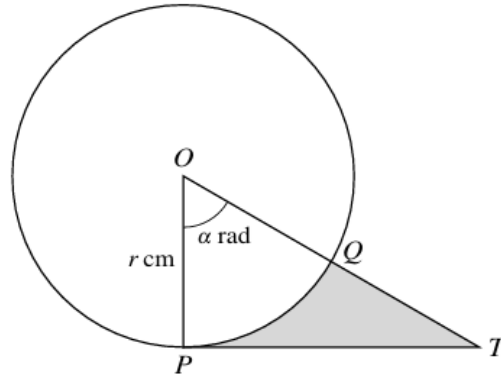
Topic 4-Circular Measure

Question No (18)

<http://kingcambridgesolutions.com>

WhatsApp +923454231525

Rs:300/Paper

Question No (18)

The diagram shows a circle with radius r cm and centre O . The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q .

- (i) Express the perimeter of the shaded region PQT in terms of r and α .
- (ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures.

Solution

On Next page

From the
 $\triangle OPT$

$$\tan \alpha = \frac{\text{vertical}}{\text{base}}$$

$$= \frac{PT}{OP}$$

$$\tan \alpha = \frac{PT}{r}$$

$$PT = r \tan \alpha$$

and

$$\cos \alpha = \frac{\text{base}}{\text{hypotenuse}}$$

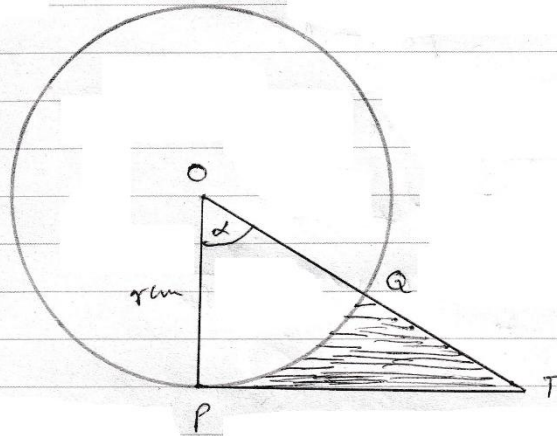
$$\cos \alpha = \frac{OP}{OT}$$

$$\cos \alpha = \frac{r}{OT}$$

$$OT = \frac{r}{\cos \alpha}$$

$$\therefore QT = OT - OQ$$

$$QT = \frac{r}{\cos \alpha} - r$$



$$\text{Arc length } \widehat{PQ} = r\alpha$$

$$= s = r\alpha$$

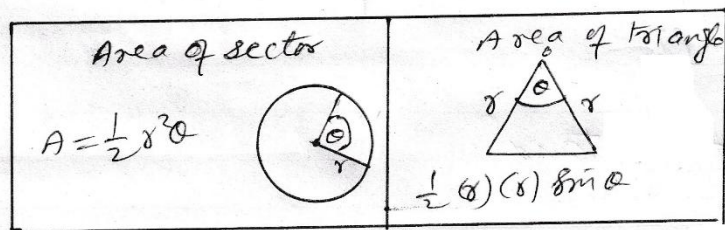
$$\widehat{PQ} = r\alpha$$

\therefore perimeter of the shaded region

$$= \widehat{PQ} + PT + QT$$

$$= r\alpha + r \tan \alpha + \frac{r}{\cos \alpha} - r$$

(ii)



Area of shaded region

$$= \text{Area of triangle } OPT - \text{area of sector } OPQ$$

$$= \frac{1}{2} (r)(r \tan \alpha) - \frac{1}{2} r^2 \alpha$$

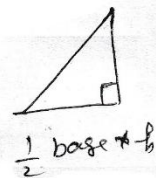
$$= \frac{1}{2} r^2 \tan \alpha - \frac{1}{2} r^2 \alpha$$

$$= \frac{1}{2} r^2 (\tan \alpha - \alpha)$$

$$= \frac{1}{2} (10)^2 (\tan \frac{\pi}{3} - \frac{\pi}{3})$$

$$= \frac{10^2}{2} \left(\sqrt{3} - \frac{\pi}{3} \right)$$

$$= 34 \text{ cm}^2$$



$$= 34 = \frac{\pi}{3}$$

