Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 4-Circular Measure

Question No (22)

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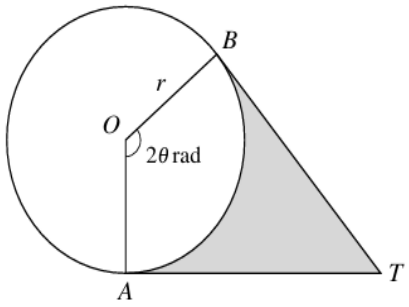

The diagram shows points A and B on a circle with centre O and radius r. The tangents to the circle at A and B meet at T. The shaded region is bounded by the minor arc AB and the lines AT and BT. Angle AOB is 2θ radians.

(i) In the case where the area of the sector AOB is the same as the area of the shaded region, show that $\tan\theta = 2\theta$.

(ii) In the case where $r = 8$ cm and the length of the minor arc AB is 19.2 cm, find the area of the shaded region.

Solution

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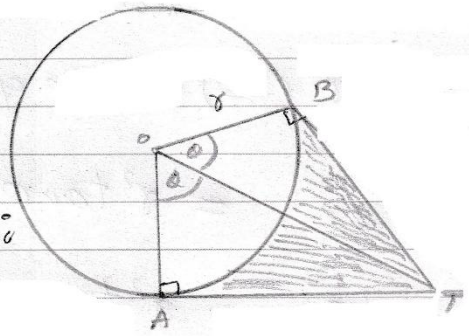
As BT is tangent at B, so

$\triangle OBT$ is right angled at $O\hat{B}T = 90^\circ$



$$\tan\theta = \frac{\text{vertical}}{\text{Base}}$$

$$\tan\theta = \frac{BT}{r}$$

$$BT = r\tan\theta$$

Area of triangle $OBT = \frac{1}{2}$ base $\times$ height

$$= \frac{1}{2} \times OB \times BT$$

$$= \frac{1}{2} \times r \times r\tan\theta$$

$$= \frac{1}{2} r^2 \tan\theta$$

As ~~Area~~ A OBT is a bite

$\Rightarrow$ Area of $AOBT = 2$(Area of $\triangle OBT$)

$$= 2\left(\frac{1}{2} r^2 \tan\theta\right)$$

$$= r^2 \tan\theta$$

By given condition

Area of sector AOB = Area of shaded region

$\frac{1}{2}r^2(2\theta)$ = Area of AOBT − area of sector AOB

$= r^2\tan\theta - \frac{1}{2}r^2(2\theta)$

$r^2\theta = r^2\tan\theta - r^2\theta$

$r^2\theta + r^2\theta = r^2\tan\theta$

$2r^2\theta = r^2\tan\theta$

$\tan\theta = 2\theta$ proved

(ii)

As given minor arc AB = 19.2 cm

As $s = r\theta$

$\Rightarrow 19.2 = 8(2\theta)$ ∵ $r = 8, \theta = 2\theta$

$\theta = \frac{19.2}{16}$

$\theta = 1.2$ rad

Area of shaded region

$= r^2\tan\theta - r^2\theta$ part (i)

$= (8)^2\tan 1.2 - (8)^2(1.2)$

$= 87.8\ cm^2$