

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

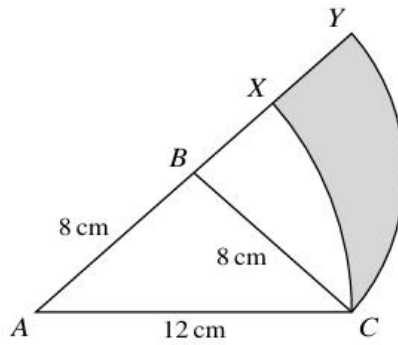
Topic 4-Circular Measure

Question No (23)

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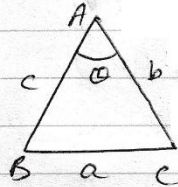
The diagram shows an isosceles triangle ACB in which $AB = BC = 8$ cm and $AC = 12$ cm. The arc XC is part of a circle with centre A and radius 12 cm, and the arc YC is part of a circle with centre B and radius 8 cm. The points A , B , X and Y lie on a straight line.

- (i) Show that angle $CBY = 1.445$ radians, correct to 4 significant figures.
- (ii) Find the perimeter of the shaded region.

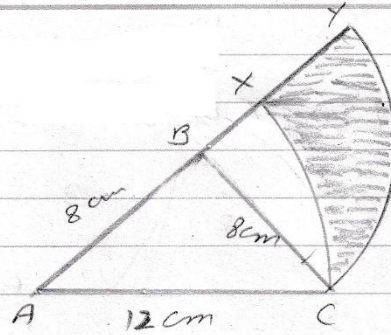
Solution

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cosine Rule



$$(a)^2 = b^2 + c^2 - 2bc \cos \theta$$



in $\triangle ABC$

$$(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC) \cos \widehat{ABC}$$

$$(12)^2 = (8)^2 + (8)^2 - 2(8)(8) \cos \widehat{ABC}$$

$$144 = 64 + 64 - 128 \cos \widehat{ABC}$$

$$144 = 128 - 128 \cos \widehat{ABC}$$

$$128 \cos \widehat{ABC} = 128 - 144$$

$$\cos \widehat{ABC} = -\frac{16}{128}$$

$$\widehat{ABC} = \cos^{-1}\left(-\frac{16}{128}\right)$$

$$\widehat{ABC} = 1.696 \text{ radians}$$

$$\widehat{CBA} = \pi - \widehat{ABC}$$

$$= \pi - 1.696$$

$$\widehat{CBY} = 1.445 \text{ radians.}$$

(ii)

$$\begin{aligned} \text{Length of arc } \widehat{YC} &= r\theta \\ &= (BC)(\widehat{CBY}) \\ &= 8(1.445) \\ &= 11.56 \end{aligned}$$

In $\triangle ABC$

$$\widehat{ABC} + \widehat{BAC} + \widehat{BCA} = \pi$$

 $\widehat{BAC} = \widehat{BCA}$ due to isosceles triangle

$$\Rightarrow 2\widehat{BAC} = \pi - \widehat{ABC}$$

$$\widehat{BAC} = \frac{\pi - \widehat{ABC}}{2}$$

$$= \frac{\pi - 1.696}{2}$$

$$\therefore \widehat{ABC} = 1.696$$

$$\widehat{BAC} = 0.7225 \text{ radians}$$

$$\therefore \text{arc } \widehat{CX} = r\theta$$

$$= 12 \times \widehat{BAC}$$

$$= 12 \times 0.7225$$

$$= 8.67 \text{ cm}$$

Perimeter of shaded region

$$= XX + \widehat{YC} + \widehat{CX}$$

$$= 4 + 11.56 + 8.67$$

$$= 24.2 \text{ cm}$$

$$Y = AC = 12$$

$$AB + BX = AC$$

$$8 + BX = 12$$

$$BX = 4$$

$$\Rightarrow XX = 4$$

