

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

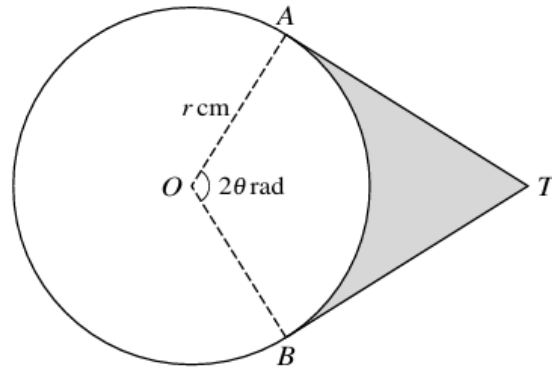
Topic 4-Circular Measure

Question No (25)

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Question No (25)

The diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and angle $AOB = 2\theta$ radians. The tangents to the circle at A and B meet at T .

- (i) Express the perimeter of the shaded region in terms of r and θ .
- (ii) In the case where $r = 5$ and $\theta = 1.2$, find the area of the shaded region.

Solution

On Next page

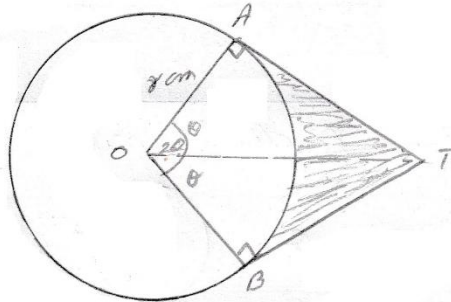
$$\text{As } \angle OAT = 90^\circ$$

From $\triangle OAT$

$$\tan \theta = \frac{\text{vertical}}{\text{Base}}$$

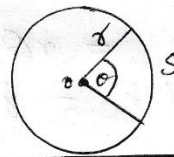
$$\tan \theta = \frac{AT}{r}$$

$$AT = r \tan \theta$$



Arc length

$$S = r\theta$$



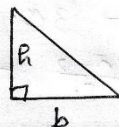
$$\begin{aligned} \text{Arc length } \widehat{AB} &= r(2\theta) \\ &= 2r\theta \end{aligned}$$

$$\begin{aligned} \therefore \text{perimeter of the shaded region} &= \widehat{AB} + AT + BT \\ &= 2r\theta + r \tan \theta + r \tan \theta \\ &= 2r\theta + 2r \tan \theta \\ &= 2r(\theta + \tan \theta) \quad \text{Answer} \end{aligned}$$

(ii)

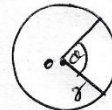
Area of triangle

$$\Delta = \frac{1}{2} \text{ base} \times \text{height}$$



Area of Sector

$$A = \frac{1}{2} r^2 \theta$$



$$\text{Area of Shaded region} = \text{Area of } \triangle OAT + \text{Area of } \triangle OBT - \text{Area of sector OAB}$$

$$= \frac{1}{2} (OA)(AT) + \frac{1}{2} (OB)(BT) - \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (8)(8 \tan \theta) + \frac{1}{2} (8)(8 \tan \theta) - \frac{1}{2} 8^2 \theta$$

$$= \frac{1}{2} (5)(5 \tan 1.2) + \frac{1}{2} (5)(5 \tan 1.2) - \frac{1}{2} (5)^2 (2(1.2))$$

$$= 32.152 + 32.152 - 30$$

$$= 34.3 \text{ cm}^2$$