

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

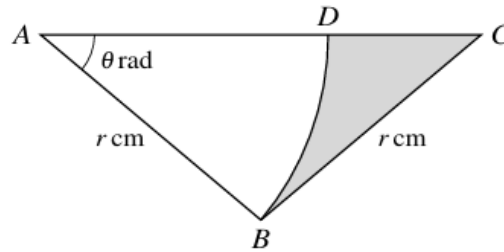
Topic 4-Circular Measure

Question No (27)

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Question No (27)

In the diagram, ABC is an isosceles triangle with $AB = BC = r \text{ cm}$ and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A .

- (a) Express the area of the shaded region in terms of r and θ .
- (b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region.

Solution

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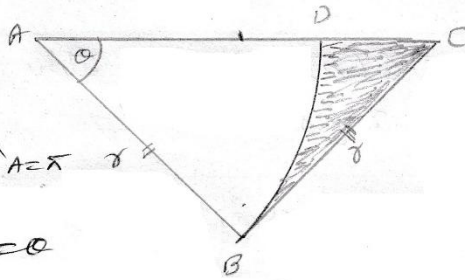
From diagram

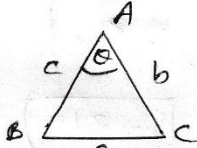
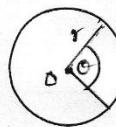
$$\hat{A}BC + \hat{B}AC + \hat{C}BA = \pi$$

$$\text{As } \hat{B}AC = \hat{C}BA = \theta$$

$$\Rightarrow \hat{A}BC + \theta + \theta = \pi$$

$$\hat{A}BC = \pi - 2\theta$$



Area of triangle	Area of sector
$\Delta = \frac{1}{2} bc \sin A$ 	$A = \frac{1}{2} r^2 \theta$ 

Area of shaded region = Area of $\triangle ABC$ - area of sector $\widehat{ABD}$

$$= \frac{1}{2} (AB)(BC) \sin(\pi - 2\theta) - \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (r)(r) \sin(\pi - 2\theta) - \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} r^2 \sin(\pi - 2\theta) - \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} r^2 \sin 2\theta - \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} r^2 (\sin 2\theta - \theta) \quad \underline{\underline{Ans}}$$

$$= \sin(\pi - 2\theta)$$

$$= \sin 2\theta$$

(b) Given

$$r = 10, \theta = 0.6$$

Diagram from (a)

$$\text{Arc length } \widehat{BD} = r\theta$$

$$= 10(0.6)$$

$$= 6 \text{ cm}$$

In $\triangle ABM$

$$\frac{AM}{AB} = \cos \theta$$

$$\frac{AM}{r} = \cos \theta$$

$$AM = r \cos \theta$$

$$\text{As } AC = 2AM$$

$$AC = 2r \cos \theta$$

$$\therefore CD = AC - AD$$

$$CD = 2r \cos \theta - r$$

$$= 2(10) \cos 0.6 - 10$$

$$CD = 6.51 \text{ cm}$$

perimeter of Shaded region

$$= \widehat{BD} + BC + DC$$

$$= 6 + 10 + 6.51$$

$$= 22.5 \text{ cm} \quad \underline{\underline{\text{Ans}}}$$

