Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (2)

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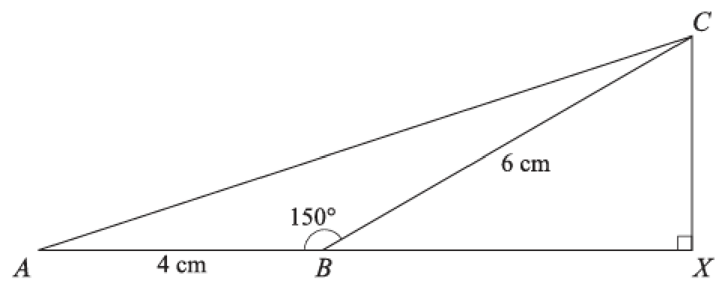

In the diagram, ABC is a triangle in which $AB = 4$ cm, $BC = 6$ cm and angle $ABC = 150^\circ$. The line CX is perpendicular to the line ABX.

(i) Find the exact length of BX and show that angle $CAB = \tan^{-1}\left(\frac{3}{4 + 3\sqrt{3}}\right)$.

(ii) Show that the exact length of AC is $\sqrt{(52 + 24\sqrt{3})}$ cm.

Solution

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(i) From the diagram

$C\hat{B}X = 180 - 150$

$C\hat{B}X = 30^\circ$

From the $\triangle BCX$

$\cos 30 = \frac{BX}{BC}$

$\cos 30 = \frac{BX}{6}$

$BX = 6 \cos 30$

$BX = 6\left(\sqrt{3}/2\right) = 3\sqrt{3}$

Now

$\frac{CX}{BC} = \sin 30$

$\frac{CX}{6} = \sin 30$

$CX = 6 \sin 30$

$CX = 6\left(1/2\right) = 3$

Now from $\triangle ACX$

$\tan C\hat{A}B = \frac{CX}{AX}$

$= \frac{3}{4 + BX}$

$\tan C\hat{A}B = \frac{3}{4 + 3\sqrt{3}}$

$C\hat{A}B = \tan^{-1}\left(\frac{3}{4 + 3\sqrt{3}}\right)$ proved

(ii) From ΔACX

using pythagoras Theorem

$(AC)^2 = (AX)^2 + (CX)^2$

$= (4 + 3\sqrt{3})^2 + (3)^2$

$= (4)^2 + 2(4)(3\sqrt{3}) + (3\sqrt{3})^2 + 9$

$= 16 + 24\sqrt{3} + 9(3) + 9$

$= 16 + 24\sqrt{3} + 27 + 9$

$(AC)^2 = 52 + 24\sqrt{3}$

$AC = \sqrt{52 + 24\sqrt{3}}$ proved