

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (8)

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Question No (8)

(i) Show that the equation $2 \tan^2 \theta \cos \theta = 3$ can be written in the form $2 \cos^2 \theta + 3 \cos \theta - 2 = 0$.

(ii) Hence solve the equation $2 \tan^2 \theta \cos \theta = 3$, for $0^\circ \leq \theta \leq 360^\circ$.

Solution

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(i) Given equation

$$2 \tan^2 \theta \cos \theta = 3$$

$$2 \frac{\sin^2 \theta}{\cos \theta} \cdot \cos \theta = 3$$

$$2 \frac{\sin^2 \theta}{\cos \theta} = 3$$

$$2 \sin^2 \theta = 3 \cos \theta$$

$$2(1 - \cos^2 \theta) = 3 \cos \theta$$

$$2 - 2 \cos^2 \theta = 3 \cos \theta$$

$$2 \cos^2 \theta + 3 \cos \theta - 2 = 0 \quad \text{proved}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin^2 \theta = 1 - \cos^2 \theta$$

(ii)

$$2 \tan^2 \theta \cos \theta = 3 \quad 0 \leq \theta \leq 36^\circ$$

$$2 \cos^2 \theta + 3 \cos \theta - 2 = 0 \quad \text{from (i)}$$

By factorization

$$2 \cos^2 \theta + 4 \cos \theta - \cos \theta - 2 = 0$$

$$2 \cos \theta (\cos \theta + 2) - 1 (\cos \theta + 2) = 0$$

$$(\cos \theta + 2)(2 \cos \theta - 1) = 0$$

$$\cos \theta + 2 = 0$$

$$\cos \theta = -2 \quad \text{ignore}$$

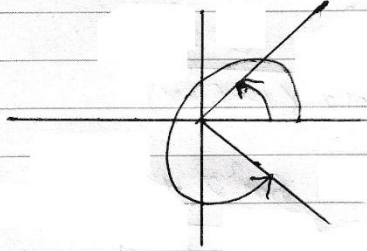
due to

$$-1 \leq \cos \theta \leq 1$$

$$2 \cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2}$$

$\cos$ is +ve in I and IV quadrant



Basic angle

$$\cos \alpha = \frac{1}{2}$$

$$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\alpha = 60^\circ$$

$$\therefore \theta = \alpha, 360 - \alpha$$

$$= 60^\circ, 360 - 60$$

$$\therefore \theta = 60^\circ, 300$$