Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (15)

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The function $f : x \mapsto 4 - 3\sin x$ is defined for the domain $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $f(x) = 2$.

(ii) Sketch the graph of $y = f(x)$.

(iii) Find the set of values of k for which the equation $f(x) = k$ has no solution.

The function $g : x \mapsto 4 - 3\sin x$ is defined for the domain $\frac{1}{2}\pi \leqslant x \leqslant A$.

(iv) State the largest value of A for which g has an inverse.

(v) For this value of A, find the value of $g^{-1}(3)$.

Solution

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$f: x \to 4 - 3\sin x$ for $0 \le x \le 2\pi$

$f(x) = 4 - 3\sin x$

(i)

$f(x) = 4 - 3\sin x$

$2 = 4 - 3\sin x$ $\quad \because f(x) = 2$ given

$3\sin x = 4 - 2$

$3\sin x = 2$

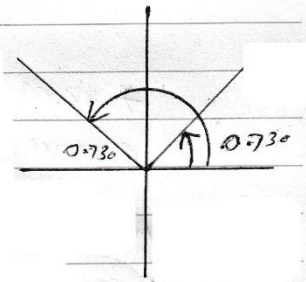
$\sin x = 2/3$

Sine is +ve in I and II quadrant.

Basic angle

$\alpha = \sin^{-1}(2/3)$

$= 0.730$ radian



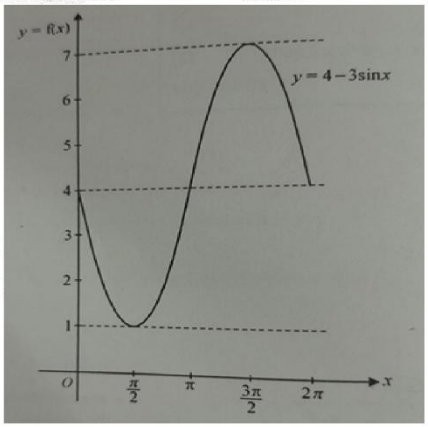
$\therefore x = \alpha, \quad \pi - \alpha$

$= 0.73^{\circ}, \quad \pi - 0.73^{\circ}$

$x = 0.730, 2.41$ radians

(i)

x	0	$\pi/2$	π	$3\pi/4$	2π
$y=f(x)$	4	1	4	7	4



(ii) From the graph we see that the curve has range $1 \le y \le 7$

Hence, the set of values of k for which $f(x) = k$ has no solution are

$k < 1$ or $k > 7$.

(iv) As $g(x) = 4 - 3\sin x$ for $\pi/2 \le x \le A$

From the graph, the largest value of A in the given domain for which g has an inverse is $3\pi/2$.

(v) $g(x) = 4 - 3\sin x$

Let $g^{-1}(3) = x$

$\Rightarrow g(x) = 3$ $\quad \because g(x) = 4 - 3\sin x$

$\Rightarrow 4 - 3\sin x = 3$

$-3\sin x = 3 - 4$

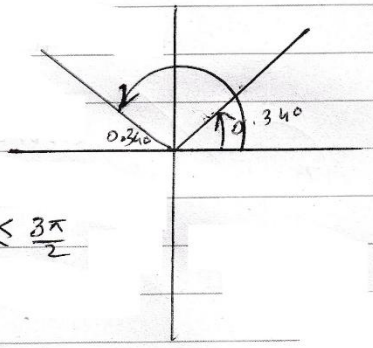
$\sin x = 1/3$

Sine is +ve in I and II quadrant

Basic angle

$\alpha = \sin^{-1}(1/3)$

$\alpha = 0.340$



Given range is $\pi/2 \leq x \leq \frac{3\pi}{2}$

$\Rightarrow x = \pi - 0.340$

$= 2.80$ radians.

$\therefore g^{-1}(3) = 2.80$ radians $\quad \because x = g^{-1}(3)$