

Cambridge International AS & A Level

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Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (18)

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**Question No (18)**

The function  $f$  is such that  $f(x) = 3 - 4\cos^k x$ , for  $0 \leq x \leq \pi$ , where  $k$  is a constant.

- (i) In the case where  $k = 2$ ,
  - (a) find the range of  $f$ ,
  - (b) find the exact solutions of the equation  $f(x) = 1$ .
- (ii) In the case where  $k = 1$ ,
  - (a) sketch the graph of  $y = f(x)$ ,
  - (b) state, with a reason, whether  $f$  has an inverse.

**Solution**

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$$f(x) = 3 - 4 \cos^k x \quad \text{for } 0 \leq x \leq \pi$$

(i) (a)

$$f(x) = 3 - 4 \cos^k x$$

when  $k=2$

$$f(x) = 3 - 4 \cos^2 x$$

$$\text{As } -1 \leq \cos x \leq 1$$

By trigono

property

$$\Rightarrow 0 \leq \cos^2 x \leq 1$$

$\therefore$  for even power min value is 0

$$0 \geq -4 \cos^2 x \geq -4$$

$$\text{or } -4 \leq -4 \cos^2 x \leq 0$$

Adding 3

$$-4 + 3 \leq 3 - 4 \cos^2 x \leq 0 + 3$$

$$-1 \leq 3 - 4 \cos^2 x \leq 3$$

$$-1 \leq f(x) \leq 3$$

$$\therefore f(x) = 3 - 4 \cos^2 x$$

$\therefore$  range of  $f$  is  $-1 \leq f(x) \leq 3$

⑥

$$f(x) = 3 - 4 \cos^2 x$$

$$\Rightarrow 1 = 3 - 4 \cos^2 x$$

$$1 - 3 = -4 \cos^2 x$$

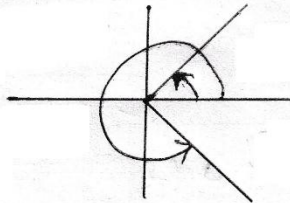
$$-2 = -4 \cos^2 x$$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \frac{1}{\sqrt{2}}$$

$$\cos x = \frac{1}{\sqrt{2}}$$

$\cos$  is +ve in  
I and IV quadrant



Basic angle

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\alpha = \pi/4$$

$$\therefore x = \alpha, 2\pi - \alpha$$

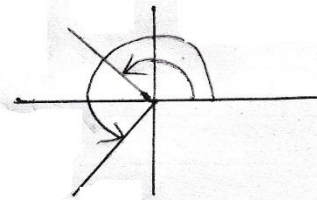
$$x = \pi/4, 2\pi - \pi/4$$

$$x = \pi/4, 7\pi/4$$

$$x = \pi/4, 7\pi/4 \text{ out of range}$$

$$\cos x = -\frac{1}{\sqrt{2}}$$

$\cos$  is -ve in II  
and III quadrant



Basic angle

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\alpha = \pi/4$$

$$x = \pi - \alpha$$

$$= \pi - \pi/4$$

$$= 3\pi/4$$

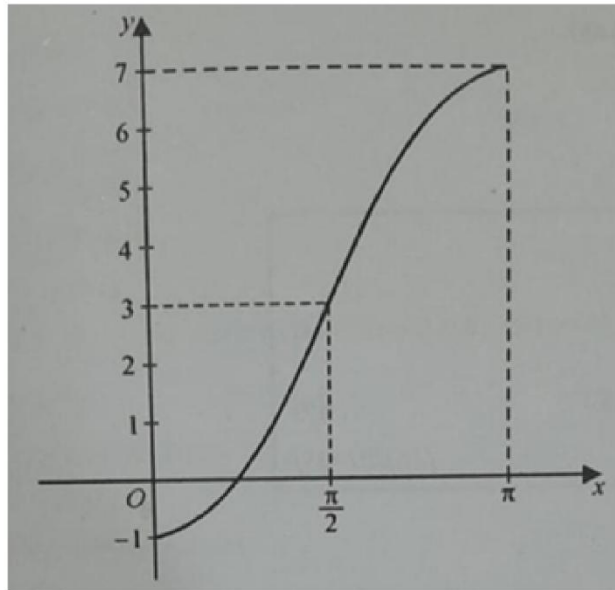
$$\therefore x = \pi/4, 3\pi/4 \quad \text{Ans}$$

(ii)

when,  $k=1$

$$f(x) = 3 - 4 \cos x$$

$$k=1$$



(b)  $f$  has an inverse because horizontal line test pass. It means that it is 1-1 function.