

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (20)

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Question No (20)

(i) Prove the identity $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$.

(ii) Solve the equation $\frac{2}{\sin x \cos x} = 1 + 3 \tan x$, for $0^\circ \leq x \leq 180^\circ$.

Solution

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(i)

$$L.H.S = \tan x + \frac{1}{\tan x}$$

$$= \frac{\sin x}{\cos x} + \frac{1}{\frac{\sin x}{\cos x}}$$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$= \frac{1}{\sin x \cos x}$$

$$= R.H.S$$

$$\sin^2 x + \cos^2 x = 1$$

(ii)

$$\frac{2}{\sin x \cos x} = 1 + 3 \tan x$$

$$0 \leq x \leq 180$$

$$2 \left(\tan x + \frac{1}{\tan x} \right) = 1 + 3 \tan x \quad \because \frac{1}{\sin x \cos x} = \tan x + \frac{1}{\tan x}$$

$$2 \tan x + \frac{2}{\tan x} = 1 + 3 \tan x$$

multiplying by $\tan x$

$$2 \tan^2 x + 2 = \tan x + 3 \tan^2 x$$

$$3 \tan^2 x + \tan x - 2 \tan^2 x - 2 = 0$$

$$\tan^2 x + \tan x - 2 = 0$$

Factorize

$$\tan^2 x + 2 \tan x - \tan x - 2 = 0$$

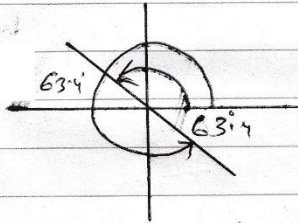
$$\tan x (\tan x + 2) - 1 (\tan x + 2) = 0$$

$$(\tan x + 2)(\tan x - 1) = 0$$

$$\tan x + 2 = 0$$

$$\tan x = -2$$

$\tan$ is -ve in II
and IV quadrant



Basic angle

$$\begin{aligned} x &= \tan^{-1}(2) \\ &= 63.4^\circ \end{aligned}$$

$$x = 180 - 63.4^\circ$$

$$x = 116.6^\circ$$

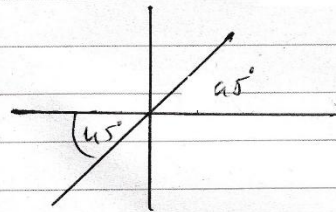
(Second angle out of $0 \leq x \leq 180$)

$$\therefore x = 45^\circ, 116.6^\circ$$

$$\tan x = 1$$

$$x = \tan^{-1}(1)$$

$$x = 45^\circ$$



angle in II quadrant
out of range for
 $0 \leq x \leq 180^\circ$

