

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (21)

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Question No (21)

- (i) Show that the equation $2 \cos x = 3 \tan x$ can be written as a quadratic equation in $\sin x$.
- (ii) Solve the equation $2 \cos 2y = 3 \tan 2y$, for $0^\circ \leq y \leq 180^\circ$.

Solution

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$$\textcircled{i} \quad 2 \cos x = 3 \tan x$$

$$2 \cos x = 3 \left(\frac{\sin x}{\cos x} \right)$$

$$2 \cos x (\cos x) = 3 \sin x$$

$$2 \cos^2 x = 3 \sin x$$

$$2(1 - \sin^2 x) = 3 \sin x$$

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \cos^2 x &= 1 - \sin^2 x \end{aligned}$$

$$2 - 2 \sin^2 x = 3 \sin x$$

$$2 \sin^2 x + 3 \sin x - 2 = 0 \quad \text{as required.}$$

$\textcircled{ii}$

$$2 \cos 2y = 3 \tan 2y$$

$$\text{put } 2y = x$$

$$\Rightarrow 2 \cos x = 3 \tan x$$

$$2 \sin^2 x + 3 \sin x - 2 = 0 \quad \text{from part (i)}$$

Factorizing

$$2 \sin^2 x + 4 \sin x - \sin x - 2 = 0$$

$$2 \sin x (\sin x + 2) - 1 (\sin x + 2) = 0$$

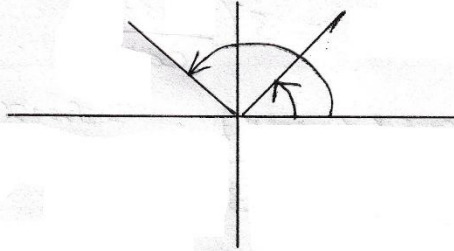
$$(\sin x + 2)(2 \sin x - 1) = 0$$

$$\begin{aligned} \sin x + 2 &= 0 \\ \sin x &= -2 \quad (\text{ignore}) \\ \therefore -1 &\leq \sin x \leq 1 \end{aligned}$$

$$\begin{aligned} 2 \sin x - 1 &= 0 \\ \sin x &= 1/2 \end{aligned}$$

$$\sin x = \frac{1}{2}$$

Sine is +ve in I and II quadrant



Basic angle

$$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

$$= 30^\circ$$

$$x = \alpha, 180 - \alpha$$

$$= 30^\circ, 180 - 30^\circ$$

$$x = 30^\circ, 150^\circ$$

$$2y = 30^\circ, 150^\circ$$

$$= x = 2y$$

$$y = 15^\circ, 75^\circ \text{ answer.}$$