

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (26)

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Question No (26)

(i) Show that the equation $1 + \sin x \tan x = 5 \cos x$ can be expressed as

$$6 \cos^2 x - \cos x - 1 = 0.$$

(ii) Hence solve the equation $1 + \sin x \tan x = 5 \cos x$ for $0^\circ \leq x \leq 180^\circ$.

Solution

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Given equation

$$1 + 8 \sin x \tan x = 5 \cos x$$

$$1 + \sin x \left(\frac{\sin x}{\cos x} \right) = 5 \cos x$$

$$1 + \frac{\sin^2 x}{\cos x} = 5 \cos x$$

$$\frac{\cos x + \sin^2 x}{\cos x} = 5 \cos x$$

$$\cos x + \sin^2 x = 5 \cos^2 x$$

$$\cos x + (1 - \cos^2 x) = 5 \cos^2 x \quad \because \sin^2 x = 1 - \cos^2 x$$

$$\cos x + 1 = 5 \cos^2 x + \cos^2 x$$

$$\cos x + 1 = 6 \cos^2 x$$

$$6 \cos^2 x - \cos x - 1 = 0 \quad \text{proved.}$$

(20)

$$1 + 8 \sin x \tan x = 5 \cos x \quad 0 \leq x \leq 180^\circ$$

$$6 \cos^2 x - \cos x - 1 = 0 \quad \text{from part (i)}$$

by factorization

$$6 \cos^2 x - 3 \cos x + 2 \cos x - 1 = 0$$

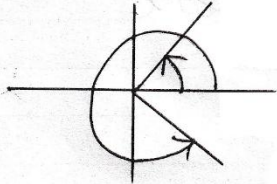
$$3 \cos x (2 \cos x - 1) + 1 (2 \cos x - 1) = 0$$

$$(2 \cos x - 1) (3 \cos x + 1) = 0$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$\cos$ is +ve in I
and IV quadrant



$$\alpha = \cos^{-1}(1/2)$$

$$\alpha = 60^\circ$$

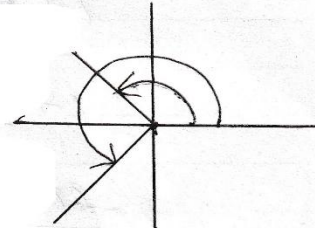
$$x = \alpha, \quad (360^\circ - \alpha \text{ is out of range due to } 0 \leq x \leq 180^\circ)$$

$$x = 60^\circ$$

$$3 \cos x + 1 = 0$$

$$\cos x = -\frac{1}{3}$$

$\cos$ is -ve in II and III quadrant



Basic angle

$$\alpha = \cos^{-1}(1/3)$$

$$\alpha = 70.5^\circ$$

$$x = 180^\circ - \alpha$$

$$= 180^\circ - 70.5^\circ$$

$$x = 109.5^\circ$$

($180^\circ + \alpha$ out of range for $0 \leq x \leq 180^\circ$)

$$\therefore x = 60^\circ, 109.5^\circ$$

